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Forecasting Some Selected Macroeconomic Variables with BVAR Models under Natural Conjugate Prior.

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Abstract. Bayesian VARs are mostly used in computational analysis of macroeconomic variables of a nation. The natural conjugate prior when combined with the likelihood function gives a posterior distribution that belong to the same distributional family. The study investigates the forecasting of some selected macroeconomic variables with BVAR Model using special types of natural conjugate prior called the symmetric and asymmetric natural conjugate prior. The data for the macroeconomic variables were obtained from the statistical bulletin of the Central Bank of Nigeria (CBN) ranging from 1986 to 2019. The forecasting assessment used is the Root Mean Square Factor Error (RMSFE). The RMSFE with small value indicates a better forecast performance. Forecasting the Macroeconomic variables with BVAR under the natural conjugate prior (symmetric and asymmetric natural conjugate prior), it was discovered from the study that asymmetric natural conjugate Prior is the best natural conjugate prior that should be used in forecasting macroeconomic variables of developing country like Nigeria. There is an inverse relationship between unemployment rate and other selected macroeconomic variables used in this study. Therefore, the policy makers should endeavor to formulate policies that will reinvigorate the economy so that single–digit inflation rate can be achieved and unemployment rate will be reduced to the barest minimum. This will help to boost the GDP and economy at large.

Keywords: BVAR, Macroeconomic Variables, Natural Conjugate Prior, Optimal Hyperparameter.

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1. Introduction

Bayesian VARs are mostly used in computational analysis of macroeconomic variables of a nation. As it is replete in the literature, it is the workhorse of econometrical analysis. The application of Bayesian analysis to macroeconomic

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variables came to limelight after the pioneered forecasting work by Litterman (1979) and Doan *et al.* (1984). It was after this that VARs and BVARs assumed the prime position of standard macroeconomic tools use for forecasting and structural analysis by econometricians and policy makers. BVAR model contains large parameters and they are treated as random variables. Bayesian influences from BVAR lend an opportunity to update probability distributions of unobserved parameters which is conditioned on the observed data (Banbura *et al.*, 2010).
According to Verde (2016), the popularity of Bayesian econometrics can be attributed to the ability of Bayesian methods to combine multi sources of information in a single model. Also, because of the availability of Bayesian statistical software, construction of complex phenomenon is easily done and this has led to computational evolution in Bayesian econometrics. Computation in Bayesian econometrics involves large data and it requires large BVARs. For optimal application of BVAR models, natural conjugate prior is required. The natural conjugate prior is necessary because it gives a wide range of useful analytical computation and results. It is the kronecker product structure of the natural conjugate prior that speeds up the computational analysis (Chan, 2019). The natural conjugate prior also known as the convenience prior is a type of prior when combined with the likelihood function gives a posterior distribution that belongs to the same distributional family (Koop, 2003). The natural conjugate prior and the posterior distribution possess similar functional form (Koop, 2003). The natural conjugate prior gives useful and reliable macroeconomic analytical results. Besides, with the presence of posterior covariance matrix of BVAR coefficient, it speeds up computational analysis and facilitates simulation when used with natural conjugate prior (Chan, 2019).
On the other hand, the presence of the kronecker product structure can serve as limitation to the (symmetric) natural conjugate prior (Chan, 2019). The presence of the kronecker product structure does not restrict cross-equation and requires symmetric treatment of its own lags and the lags of other variables presence in the model. In most cases, cross variable shrinkage in natural conjugate prior is not implementable because of the presence of kronecker product structure (Chan, 2019, Chan *et al.*, 2019). This new prior developed by Chan (2019) called the Asymmetric Conjugate Prior (ACP) takes care of all these limitations inherent in other priors (symmetric prior that was proposed by Carriero *et al.*, 2015). The asymmetric conjugate prior is an improvement on the other natural conjugate priors. The asymmetric natural conjugate prior permits the asymmetric treatment between own lags and the lags of other variables in the model. It also allows differential treatment between the variances of prior on own lags against lags of other variables in the models. The asymmetric conjugate prior also evaluates useful analytical results like the closed-form expression that is associated with the marginal likelihood. The expression for marginal likelihood obtained from the ACP is necessary in choosing the length of lags of BVARs.
According to Chan *et al.* (2019), the natural conjugate prior (NCP) relies on few hyperparameters which control the shrinkage but using a data-based approach in selecting the hyperparameters is more appealing because it reduces the subjective choices expected of the expert (users). It also helps substantially in improving forecast performance and structural analysis of BVARs. Also, as attested by

other scholars that the simplest way of improving the forecast performance of BVARs is by optimizing the hyperparameters and shrinking effectively the error covariance matrix (Chan, 2019). Shrinkage simply mean imposing restrictions or tightness on the parameters of the model while hyperparameters are the parameters of the prior distribution which are specified by the researcher (Chan, 2019, Carriero *et al.*, 2015).

The BVAR model is written in the structural form, in such a way that the error covariance matrix is diagonal. This involves an equation-by-equation estimation approach. We assume that the parameters are a priori independent across equations implying, the joint prior density is a product of densities, each for the set of parameters in each equation. Also, the VAR coefficients and the error variance in each equation follow a Normal-Inverse-Gamma prior (NIG). The posterior distribution has the same distributional form which means it is a product of Normal-Inverse-Gamma densities. The hyperparameters of the prior assume a standard Normal-inverse-Wishart (NIW) prior on the reduced-form error covariance matrix. This implied prior on the structural-form impacts the covariance matrix and error variance is a product of Normal-Inverse-Gamma densities. Hence, hyperparameters are elucidated in the reduced-form prior, which is often more natural, and the implied hyperparameters is obtained in the structural-form prior (Carriero *et al.*, 2018 and Chan, 2019).

Onwukwe and Nwafor (2014) opined that the three main macroeconomics of any economy in the twenty first century are Gross Domestic Product (GDP), inflation rate and unemployment rate. These variables among others when properly monitored and handled can galvanized the economic stability of a nation. In an economy where inflation, unemployment, Gross Domestic Product (GDP) are the key macroeconomic variables, the prior knowledge of event like previous macroeconomic policies, initial level of income of the citizenry, political instability, change of political regime, insecurity, natural disaster etc can be modelled into probability distribution that serve as prior information for researcher investigating the effects of these variables.

The essence of monetary policy is to maintain price stability in the economy of a nation. The Central Bank of Nigeria (CBN) uses the Monetary Policy Rate (MPR) to control inflation rate in Nigeria (Acha, 2018). In short, the main objective of MPR in Nigeria is to stabilize the main macroeconomic variables of the economy. Inflation can be seen as a continuous rise in the general price level of goods and services over a time. As unemployment, inflation has remained one of the major macroeconomic problem for the world. All nations make concerted efforts to maintain low inflation rate because it brings about substantial instability in the economy (Onwukwe and Nwafor, 2014). In addition, Unemployment is one of the main macroeconomic problem suffered by many countries. This problem is worst with the developing nations. Unemployment emanates from the inability to successfully utilize available labour assets for the development of the economy (Acha, 2018, Adenomon and Ojo, 2019).

Therefore, the purpose of the article is to elucidate the forecasting of some selected macroeconomic variables of a developing economy like Nigeria with BVAR model under the natural conjugate prior.

2. Materials and methods

The data for the macroeconomic variables were obtained from the statistical bulletin of the Central Bank of Nigeria (CBN) ranging from 1986 to 2019.

2.1 *BVAR Model*

Vector Autoregressive (VAR) Models are linearly stochastic models that explained the joint dynamism of multivariate time series. Suppose y_t be a random vector of endogenous variables with order ρ, then

$$VAR\left(\rho\right) = c + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \cdots + \rho_n y_{t-n} + \mu_t \tag{1}$$

and the standard VAR is given as;

$$y_t = \beta + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \cdots + \rho_n y_{t-n} + \mu_t \tag{2}$$

where β is a vector intercept, $\rho_1, \rho_2, \cdots, \rho_n$ are VAR coefficient matrices and $\mu_t \sim iidN\left(0, \Sigma\right)$
but

$$Vec\left(\mathbf{B}/\Sigma\right) \sim N\left[Vec(B), \Sigma \otimes V\right] \tag{3}$$

where $\otimes$ is the kronecker product structure, $\mathbf{B} = \left(\beta, \rho_1, \rho_2, \cdots, \rho_n\right)'$
The likelihood function of the VAR model in (1) which is a function of multiples of Gaussian densities is given as (4) below:

$$\rho(y \mid \theta, \sigma^2) = \prod_{i=1}^{n} \rho(y_i \mid \theta_i, \sigma_i^2) = \prod_{i=1}^{n} \left(2\pi\sigma_i^2\right)^{-\frac{T}{2}} e^{-\frac{1}{2\sigma_i^2}(y_i - X_i\theta_i)'(y_i - X_i\theta_i)} \tag{4}$$

The BVAR is mainly a "restricted" form of VAR Model. The general BVAR Model by Litterman (1979) is given as;

$$Y_t = \delta_0 + \sum_{i=1}^{\rho} \beta_i R_{t-n} + \varepsilon_t \tag{5}$$

where Y_t is an $(n \times 1)$ vector of the variables at time t, β_i is an $(n \times n)$ matrix of the coefficients of the variables, ρ is the lag length and ε_t is the $(n \times 1)$ vector of errors (Carriero *et al.*, 2019 and Chan, 2019).
The BVAR model in relation to the three macroeconomic variables is given as:

$$y_t = \delta_0 + GDP_{t-\rho} + INFR_{t-\rho} + UNEM_{t-\rho} + \varepsilon_t \tag{6}$$

Where Y_t is an $(n \times 1)$ vector of the variables at time t (dependent variable), Gross Domestic Product (GDP), Inflation rate (INFR) and Unemployment rate (UNEM) are the macroeconomic variables and ε_t is the $(n \times 1)$ vector of errors.

2.2 *The Asymmetric Natural Conjugate Prior*

The Asymmetric Natural Conjugate Prior (ANCP) is special type of natural conjugate prior developed by Chan (2019) and it is the Normal-Inverse-Gamma prior for $\left(\phi_i, \sigma_i^2\right)$ such that $i = 1, 2, \cdots, n.$
The Normal-Inverse Gamma prior distribution is a type of natural conjugate prior (asymmetric conjugate prior) that permits the differential treatment of prior variances of own lags against the lags of others.
Given a pair $\left(\theta_i, \sigma^2\right)$ such that $i = 1, 2, \cdots, n$; then the NIG prior density function is as follows

$$pr\left(\theta, \sigma^2\right) = \prod_{i=1}^{n} k\left(\sigma_i^2\right)^{\left(c_i + 1 + \frac{v_i}{2}\right)} exp^{-\frac{1}{2\sigma_i^2}\left(S_i + \frac{1}{2}(\theta_i - d_i)'(\theta_i - d_i)\beta_i^{-1}(\theta_i - d_i)\right)} \tag{7}$$

where $k = (2\pi)^{-\frac{v_1}{2}} \mid \beta \mid^{-\frac{1}{2}} S_i^{c_i}/\Gamma(c_i)$.

θ_i is the prior variance, (d_i, β_i, c_i, S_i) such that $i = 1, 2, \cdots, n$; are the hyperparameters of the asymmetric conjugate prior. Also, the prior covariance matrix β_i induces shrinkage in the prior distribution.
Furthermore,

$$\left(\theta_i \mid \sigma_i^2\right) \sim N\left(d_i, \sigma_i^2, \beta_i\right), \quad \sigma_i^2 \sim IG\left(c_i, S_i\right),$$

this implies that;

$$\left(\theta_i \mid \sigma_i^2\right) \sim NIG\left(d_i, \beta_i, c_i, S_i\right) \tag{8}$$

(Chan, 2019, and Chan *et al.*, 2019).

2.3 *Posterior Distribution for Normal Inverse Gamma Prior Distribution*

Given the likelihood function of (4) and the asymmetric natural conjugate prior distribution of (7), the resultant posterior distribution is the combination of these equations which is as follows;

The posterior distribution of $\left(\theta_i, \sigma_i^2\right)$ for $i = 1, 2, ..., n$, is

$$pr\left(\theta_i, \sigma^2/y\right) \propto pr\left(\theta, \sigma^2\right) pr\left(y/\theta, \sigma^2\right)$$

$$pr\left(\theta_i, \sigma^2/y\right) = \prod_{i=1}^{n} pr\left(\theta_i, \sigma_i^2\right) pr\left(y/\theta, \sigma^2\right)$$

$$pr\left(\theta_i, \sigma^2/y\right) = \prod_{i=1}^{n} k_i (2\pi)^{-\frac{n}{2}} \left(\sigma_i^2\right)^{-\left(c_i+1+\frac{n+k_i}{2}\right)} exp^{-\frac{1}{2\sigma_i^2}\left(\bar{S}_i+\frac{1}{2}(\theta_i-\bar{\theta}_i)' K_{\theta_i}(\theta_i-\bar{\theta}_i)\right)} \tag{9}$$

where $K_{\theta_i} = \beta_I^{-1} + X_I' X_i, \qquad \bar{\theta}_i = K_{\theta_i}^{-1}\left(\beta_I^{-1} d_i + X_I^{-1} y_i\right),$

and

$$\bar{S} = S_i + \frac{\left(y_i' y_i + d_i' \beta_I^{-1} d_i - \bar{\theta}_i' K_{\theta_i}^{-1} \theta_i\right)}{2}$$

In a compact form, the posterior distribution is expressed as;

$$\left(\theta_i, \sigma_i^2 \mid y\right) \sim NIG\left(\bar{\theta}_i, K_{\theta_i}^{-1}, c_i + \frac{n}{2}, \bar{S}_i\right) \qquad fori = 1, 2, \cdots, n.$$

This shows that the posterior distribution is also a multiple Normal Inverse Gamma (n-NIG distributions). Also, from the properties of NIG distribution, the posterior mean and variance can be obtained (Koop and Kolobilis, 2010 and Chan *et al.*, 2019).

2.4 *Posterior Predictive for Normal Inverse Gamma Prior Distribution*

The posterior predictive for the Normal Inverse Gamma Prior Distribution is given as;

$$p\left(x \mid y\right) = \int\int p\left(x \mid \mu, \sigma^2\right) p\left(\mu, \sigma^2 \mid y\right) d\mu d\sigma^2$$

$$= \frac{p\left(x \mid y\right)}{p\left(y\right)}$$

$$= \frac{\Gamma[(v_n+1)/2]}{\Gamma(v_n/2)} \sqrt{\frac{k_n}{k_n+1}} \frac{(v_n \sigma_n^2)^{v_n/2}}{\left[v_n \sigma_n^2 + \frac{k_n}{k_n+1}(x-\mu_n)^2 (v_n+1)/2\right]} \frac{1}{\pi^{1/2}}$$

$$= \frac{\Gamma[(v_n+1)/2]}{\Gamma(v_n/2)} \left[\frac{k_n}{(k_n+1)\pi v_n \sigma_n^2}\right]^{\frac{1}{2}} \left[1+\frac{k_n}{(k_n+1)\pi v_n \sigma_n^2}\right]^{-(v_n+1)/2}$$

In a simplified form, the above equation becomes;

$$p(x/y) = t_{v_n}\left(\mu_n, \frac{(1+k_n)\sigma_n^2}{k_n}\right) \tag{10}$$

(Chan *et al.*, 2019).

2.5 *Marginal Likelihood of VAR under Natural Conjugate Prior*

The analytical expression of marginal likelihood helps to obtain the optimal hyperparameter of the prior distribution. The optimal hyperparameter obtained controls the tightness (shrinkage) of the prior (Chan *et al.*, 2019). For the avoidance of mathematical ambiguity, it is expedient to compute the marginal likelihood in logarithms scale. The marginal likelihood of VAR(p) is obtained by combining the likelihood function of (4) and NIG prior of (7), and the resultant marginal likelihood is given as;

$$pr(y) = (2\pi)^{-\frac{nt}{2}} \mid \Omega K_{\theta_i} \mid^{-\frac{n}{2}} \frac{\Gamma_n\left(\frac{v_0+t}{2}\right) \mid \underline{S} \mid^{\frac{v_0}{2}}}{\Gamma_n\left(\frac{v_0}{2}\right) \mid \bar{S} \mid^{\frac{v_0+t}{2}}}$$

Where K_{θ_i} is the precision matrix (Chan *et al.*, 2019).

The log marginal likelihood is;

$$\begin{aligned} logpr(y) &= -\frac{nt}{2}log(2\pi) \\ &+ \sum_{i=1}^{n}\left[-\frac{1}{2}\left(log\Omega + logK_{\theta_i}\right) + log\Gamma_n\left(\frac{v_0+t}{2}\right) + \frac{v_0}{2}log\underline{S} - log\Gamma_n\left(\frac{v_0}{2}\right) - \frac{v_0+t}{2}log \mid \bar{S} \mid\right] \end{aligned} \tag{11}$$

(Chan, 2019).

The conditional likelihood of the VAR(p) in (1) can be written in a compact form as:

$$\left(\theta_i \mid y, \sigma_i^2\right) \sim N\left(\hat{\theta}_i, \sigma_i^2 \mathbf{A}_{\theta_i}^{-1}\right)$$

where $\sigma_i^2 \mathbf{A}_{\theta_i}^{-1}$ is the covariance matrix.

2.6 *Forecast Assessment of the Model*

In this section, we look at the forecasting performance of the BVAR model under the natural conjugate priors. The optimal hyperparameters of the priors are obtained by maximization of the marginal likelihood which helps in obtaining the point and density forecast of the variables. In the evaluation, conditional expectation is used as the h-step ahead point forecast and predictive density is used as the density forecast. The metric comparison used in the evaluation of the forecast of the BVAR model is the Root Mean Squared Forecast Error (RMSFE) and it is given with horizon h as:

$$RMSFE_{i,h} = \sqrt{\frac{\sum_{t=t_0}^{T-h}\left(y_{i,t+h}^{0} - E\left(y_{i,t+h} \mid y_{1:t}\right)\right)^2}{T-h-t+1}} \tag{12}$$

Where $y_{i,t+h}$ is the h-step ahead forecast and $y_{i:t}$ is 1-step ahead forecast (Carriero, *et al.*, 2015). Under RMSFE, least values indicate robust prediction or forecasting. Also, the RMSFE of a model is computed to ascertain if there exist statistical difference (robustness) in the forecasting performance over a period (Domit *et al.*, 2019).

3. Results and Discussion

The section centres on the analysis of data. The analysis is done with the hyperparameters specified by Chan (2019) as shown in table 1 and the associated Log Marginal Likelihood,

Table 1: Optimal Values of Hyperparameter under Asymmetric prior by (Chan, 2019).

Hyperparameters	Symmetric Prior	Asymmetric Prior
k_1	0.0390	0.4060
k_1	0.0390	0.0091
Log-ML	-9436	-9201

Table 1 shows the hyperparameters (optimal) and the associated log marginal likelihood adopted from Chan (2019) with the values of symmetric prior adopted from Carriero *et al.* (2015) which are benchmark. Symmetric prior treats both own lags and lags of other variables alike while asymmetric prior shrink coefficients of lags of other variables more strongly or aggressively than its own lags. The ideal behind this is that more information are contain in the lags of own variables than that of other variables during the evaluation and estimation of macroeconomic variables (Chan, 2019).

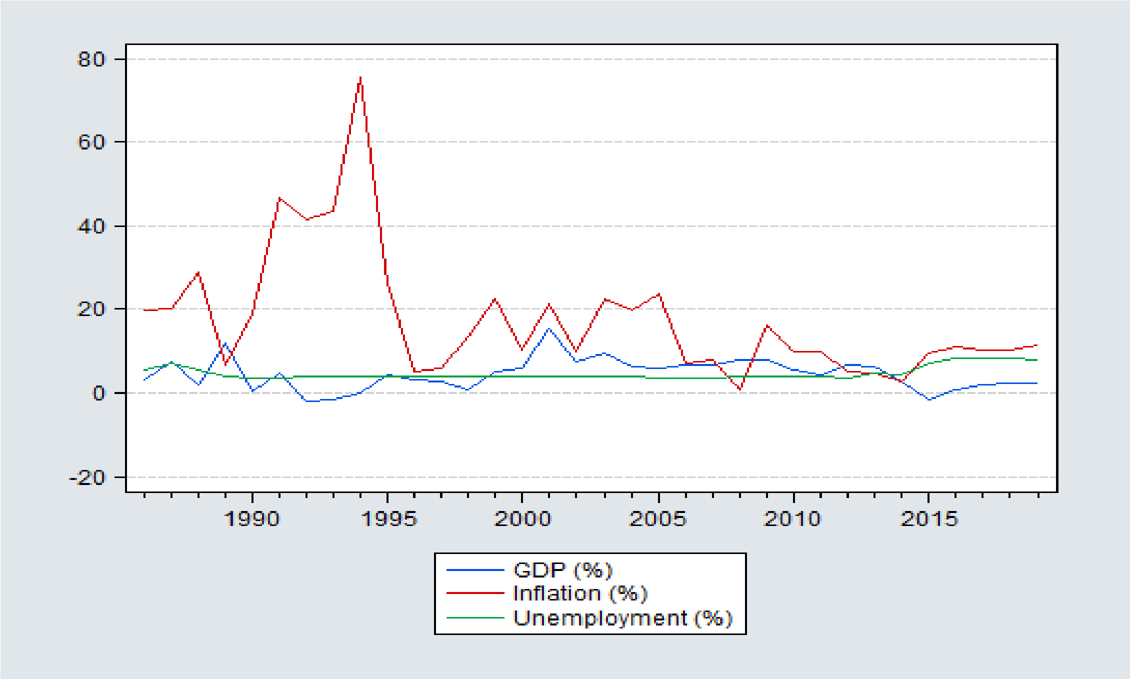

Figure 1: Graph of GDP, inflation rate and unemployment rate in Nigeria

Figure 1 shows the graph of the steady growth rate of the macroeconomic variables. The graph reveals the upward and downward movement of the macroeconomic variables. The graph also mirrors the effects of the variables under consideration within the study period. They variables are; Gross Domestic Product (GDP), inflation rate and unemployment rate.

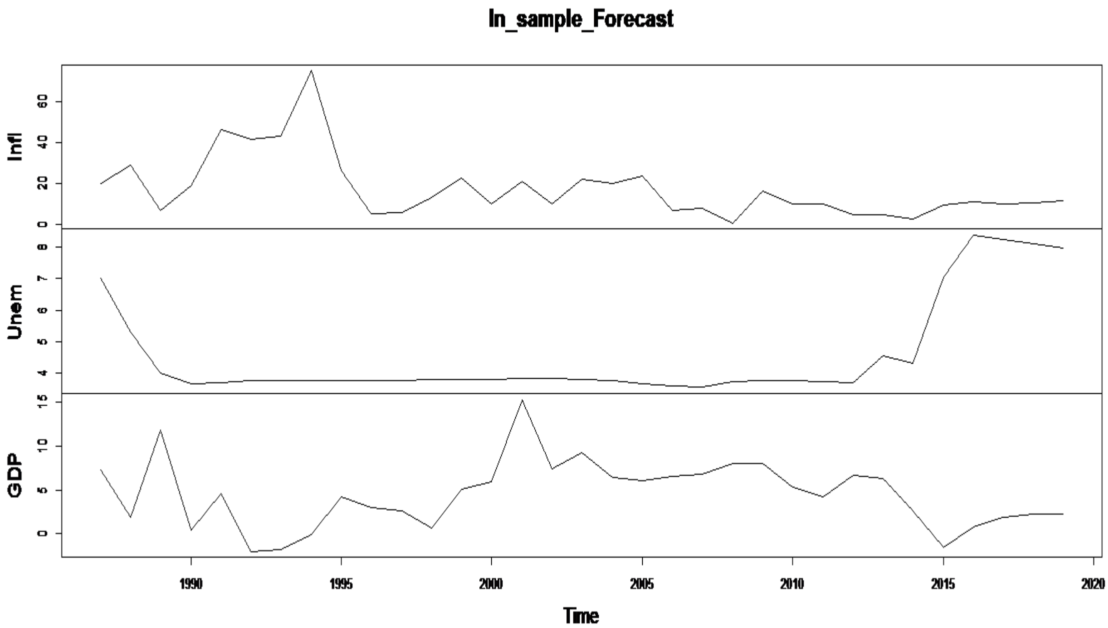

Figure 2: In-sample forecast graph for the macroeconomic variables

Figure 2 shows the in-sample forecast of the three selected macroeconomic variables. It reveals that unemployment has a smooth curve compared to inflation rate and GDP in Nigeria. From the In-sample forecast, it is evident that from 2015, there is upward movement of the curve of inflation rate and GDP growth rate respectively. The unemployment rate rises up steadily. This might be connected with the change in policies of the Government and pre-effect of Covid-19 in which the GDP growth rate of Nigeria is on a decline. There is hope as the GDP curve started rising after the adverse effect of the pandemic that affected both human and economic resources of the nation.

Table 2: Specification of Lag Length

Forecasting Assessment			
Lag	AIC	BIC	HQ
1	6.686335	7.246814	6.865637
2	7.002538	7.983377	7.316317
3	7.233837	8.635035	7.682093
4	7.595831	9.417388	8.178563

Table 2 shows the three forecasting assessment used to determine the appropriate lag length. AIC has the least values for each lag level. Also, lag(1) of AIC is least signifying the best lag level for the analysis. This is in agreement with Adenomon and Ojo (2019) assertion that one or two of annual data usually suffice in determining the optimal lag length.

Table 3: BVAR Lag Specification of the Variables

BVAR Lag Specification of the Variables				
Variables	const.	Infl. lag1	Unem. lag1	GDP lag1
Infl.	17.747339	0.0403310	-0.09627042	-0.10644848
Unem.	4.371836	7-0.001763	0.06104474	-0.01053705
GDP	4.584959	-0.003081	-0.03825159	0.02677674

In Table 3, we looked at the variables' relationship at the first lag. It can been seen from the first column that inflation rate is positively related to inflation at lag(1), negatively related to both unemployment rate and GDP at lag(1). This agreed with the assertion of the Phillip's curve law which states that there is an inverse relationship between inflation and unemployment rate of a country. In the second column, unemployment is negatively related to both inflation rate and GDP at lag(1). This agreed with the Okun's law that mirrored the inverse relationship between GDP and unemployment of a country (Adenomon and Tela, 2017). And finally, in third column, there is a reserve relationship between GDP and the other variables.

Table 4: Estimation of Posterior mean under the two priors at lag(1)

Variables	Symmetric Prior	Asymmetric Prior
Gdp-1	1.2826	1.2246
Infr-1	1.0465	1.01231
Uner-1	0.9850	0.8590
Constant	-1.1101	-1.0006

From table 4, the posterior mean of asymmetric conjugate prior has the least values. This shows that in forecasting macroeconomic variables of a nation using the natural conjugate prior, asymmetric natural conjugate prior is better than symmetric natural conjugate prior. This is in line with Chan (2019) which says that data-driven asymmetric natural conjugate prior outperforms other natural conjugate priors.

Table 5: Forecasting of GDP using the two priors under the different horizons

Horizon	Symmetric Prior	Asymmetric Prior
$h = 1$	0.532	0.599
$h = 2$	1.022	1.017
$h = 3$	0.594	0.602
$h = 4$	0.784	0.776
$h = 5$	0.952	0.710
$h = 6$	1.021	1.010
$h = 7$	0.894	0.997
$h = 8$	2.034	1.223

From Table 5, the values of symmetric prior and asymmetric priors of the natural conjugate prior are very close. But at long-run the values begin to diverge. This shows that the values obtained under the two priors when forecasting the GDP of Nigeria are almost the same at short run but produces different results at long run. Furthermore, the asymmetric natural conjugate prior has the least values under the different horizons. This also agreed with the fact that asymmetric prior do outperform the benchmark symmetric prior.

Table 6: Forecasting of Inflation Rate using the two priors under the different horizons

Horizon	Symmetric Prior	Asymmetric Prior
$h = 1$	1.532	1.599
$h = 2$	1.212	1.157
$h = 3$	1.594	1.602
$h = 4$	1.784	1.766
$h = 5$	1.952	1.710
$h = 6$	1.021	1.010
$h = 7$	2.894	2.997
$h = 8$	2.314	1.223

Table 6 shows the forecasting of inflation rate under the two different priors in h-step forecast ahead. The forecast of asymmetric conjugate prior is better than the forecast of symmetric natural conjugate prior. The asymmetric conjugate prior forecast is better because it yields a consistent result with the actual forecast of inflation rate at the different horizons. This is in line with Chan (2019) with the assertion that the asymmetric conjugate prior gives a robust forecasting output than the others because of the presence of kronecker product structure which facilitates the computational tractability of macroeconomic variables.

Table 7: Forecasting of Unemployment Rate using the two priors under the different horizons

Horizon	Symmetric Prior	Asymmetric Prior
$h = 1$	1.022	1.031
$h = 2$	1.112	1.137
$h = 3$	1.594	1.602
$h = 4$	1.784	1.766
$h = 5$	1.952	1.710
$h = 6$	1.021	1.010
$h = 7$	1.890	1.987
$h = 8$	1.304	1.223

The above Table 7 shows the forecasting of unemployment rate under the two different priors in h-step forecast ahead. The forecast of the asymmetric conjugate prior again is better and more robust than the symmetric prior. The asymmetric conjugate prior forecast is better because it yields a consistent result with the actual forecast of unemployment rate at the different horizon. The forecasting under the symmetric conjugate prior closely followed that of asymmetric prior in forecasting order. This shows that in forecasting macroeconomic variables, asymmetric natural conjugate prior gives a robust prediction than the symmetric prior. This is in line with Chan (2019).

Table 8: In-Sample Forecasting of the selected Macroeconomic Variables Using BVAR.

In-sample forecasting of the macroeconomic variables		
Inflation	Unemployment	GDP
17.68978	4.626985	4.407306
17.10706	4.686344	4.451294
18.20111	4.624081	4.344377
16.37730	4.480132	4.726832
18.11924	4.557004	4.397253
18.78665	4.464684	4.424509
19.28383	4.548246	4.258976
19.32437	4.544118	4.259255
20.43377	4.469206	4.206942
18.00568	4.511029	4.471594
17.27648	4.561466	4.504266
17.35311	4.563585	4.491700
17.86238	4.573413	4.414137
17.76337	4.509732	4.504938
17.15980	4.522439	4.567829
16.59912	4.406286	4.784283
16.99243	4.510306	4.605452
17.30003	4.466299	4.618746
17.50273	4.497280	4.553149
17.71317	4.488740	4.534092
16.98851	4.507812	4.602984
17.00637	4.502744	4.606157
16.56120	4.512988	4.655822
17.19076	4.488773	4.604887
17.21360	4.528785	4.552802
17.33927	4.537424	4.524889
16.88117	4.518677	4.606747
16.82460	4.575497	4.565135
17.16567	4.601974	4.482241
17.62487	4.803067	4.242131
17.30189	4.855867	4.251455
17.16228	4.836582	4.289657
17.15093	4.824716	4.302316
RMSFE-Infl=14.78304		
RMSFE-Unem=1.471159		
RMSFE-GDP=3.761458		

Table 8 shows the In-Sample Forecasting of the selected Macroeconomic Variables. It is used to evaluate the predictive capacity of the model in respect to the selected macroeconomic variables. The value of RMSFE of each variable gives the in-sample forecast of the variable and the forecasting assessment (RMSFE) with the least value indicates a robust forecasting. The in-sample forecast gives a better and robust forecasting of Unemployment rate having RMSFE of 1.4711590, followed by GDP (RMSFE of 3.761458) in Nigeria.

This is in agreement with Chen and Leung (2003) and George *et al.* (2018).

4. Conclusion

The study discusses the forecasting of some selected macroeconomic variables with BVAR model in Nigeria using natural conjugate prior. The essence of the research is to forecast the macroeconomic variables using the Bayesian vector autoregressive model under the symmetric natural conjugate prior and improved natural conjugate (asymmetric) prior developed by Chan (2019). From the estimation results, the study concludes that BVAR with asymmetric natural conjugate prior gives a robust and better forecasting of unemployment rate in Nigeria. Using the specified symmetric prior as the benchmark, it was revealed from the study that Asymmetric Natural Conjugate Prior is the best Natural Conjugate Prior that should be used in forecasting macroeconomic variables of a developing country like Nigeria. There is an inverse relationship between unemployment and other selected macroeconomic variables used in this study. Therefore, the policy makers should endeavor to formulate policies that will reinvigorate the economy so that single–digit inflation rate can be achieved and unemployment rate will be reduce to the barest minimum. This will help to boost the GDP and economy at large.

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