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Studying the Impacts of Inflation, Import and Export on Gross Domestic Product using Markov Switching Vector Autoregressive Model

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Abstract. Some macroeconomic variables have been studied based on their impacts and contribution to the economic growth especially in Nigeria. The study focuses on the impacts of inflation, import and export on the gross domestic product using Markov switching vector autoregressive(MS-VAR) model which is designed for econometric modeling of univariate and multiple time series subject to shifts in regime. The data spanned through 1980 to 2020 from National Bureau of Statistics and Central Bank of Nigeria respectively. The result of ADF test and Philip Perron test showed that the variables are stationary at first difference I(1) at 5% critical value. The Zivot-Andrew unit root with structural break showed that the variables are also integrated of order 1. The respective dates correspond to transition of government in 1983, structural adjustment programme in 1986, election in 1993 and a coup in 1995. Inflation is statistically significant using Lee-Strazicich test for two structural breaks. The results of cointegration test indicated that there is no long run relationship among the variables of interest without and with structural breaks. MS-VAR result showed that the probability of staying in regime 1 (that is a period of expansion) in Nigeria economy is higher than in regime 2 (that is a period or state of recession) in the nation's economy which suggests that regime 1 is more persistent than regime 2 and the period of growth in regime 1 is about 16 years while it lasted 11 years in regime 2. This implies that Nigeria economy is more stable in regime 1(expansion period) compared to that of regime 2.

Keywords: Unit root; Structural breaks; Cointegration; Regime; MS-VAR

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1. Introduction

Macroeconomic variable tends to possess the following characteristics such as unit root(s), cointegration, structural changes due to policy change, pandemic and global crisis generally. When these characteristics are present, classical regression may become invalid. In the presence of these characteristics in macroeconomic variables, Markov switching vector autoregressive model is suggested while for structural breaks or endogenous breaks, the following tests such as Sequential Dickey-Fuller test, Minimum Lagrange Multiplier (LM) (Altinay, 2005; Lumsdaine and Papell, 1997); Lee and Strazicich, 2003a, 2003b, 2004) and Zivot and Andrews test (Zivot and Andrews, 1992) are proposed because of shocks that could be permanent.

In addition, macroeconomic variables often fluctuate around high and low levels; hence an unobservable egordic Markov process and the possibility of a regime shift. One appropriate method which captures the unobservable state, the transmission from one regime to another and the duration of stay in a particular regime often ignored by the linear methods is the Markov-Switching Variance Autoregressive model (Tuaneh and Essi, 2021).

The MS-VAR model can provide a systematic ability to implementing statistical methods that can also estimate efficient and consistent parameters, detect recent changes and correct the VAR model when the regimes change (Wai, *et al.*, 2003).

The Markov-Switching Vector Autoregressive (MS-VAR) model is a non-linear model that is characterized by a data generation mechanism that is non-linear. This is done by limiting the method to be linear in a particular unobservable and discrete regime. This model was introduced by Krolzig (1997) following the Hamilton concept. The MS-VAR model also serves as a generalization of the simple finite order of the VAR model. The key concept of the model is that the observable time series vectors are reliant on an unobservable state.

The Markov Switching model is set up to achieve an unobservable state through a discrete-time and state of Markov stochastic mechanism with the transition probabilities. Also, the MS-VAR model estimates and forecast time-varying problems when a change in parameters occurs. In some empirical studies, certain parameters are conditioned on the state of the Markov chain, whereas the other parameters are allowed to be regime invariant according to Tuaneh and Essi (2021).

It is very essential to note that studying the relationship among inflation, import, export and GDP in Nigeria has majorly been based on linear time series models. The vector autoregressive model has been common in examining such relationship. But inflation, import and export may have a tendency of nonlinear behavours such as fluctuations, structural breaks, etc. and hence will require a nonlinear time series model such as MS-VAR to study their relationships (see Ismail and Isa, 2008; Cuestas and Tang, 2015; Balcilar *et al.*, 2017).

Wai, *et al.* (2015) compared the model performance between linear vetor autoregressive and Markov switching vector autoregressive models on modeling structural change in time series data and the result of their study revealed that MS-VAR models with first autoregressive order outperformed the linear VAR model.

De la Torre *et al.* (2022), tested the benefit of using Markov-Switching mod-

els to forecast the views of the 26 most traded stocks in the US in a Black-Litterman(B-L) optimal selection context. The result from the study suggested that the performances of the B-L portfolios, the SampP, and the market portfolio are statistically equal in terms of returns, their mean-variance efficiency in an ex-ante or ex-post analysis.
According to Krolzig (1997), Markov Switching Vector Autoregressive (MS-VAR) model has become increasingly prominent in applications since it is able to detect the classical business cycle phases and the difference in terms of average growth rates of the economy. MS-VAR model also can increase the reliability of analysis of the business cycle, and provide useful information in the study of economic relationship because of its properties on detecting the high-level regime. Therefore, the perception of the current state of economy can be improved (Anas and Abdel-Rahman, 2004). While in the MS-VAR model approach, an exploration on the correlated Markov chain can be used to analyze the relationship between multiple time series variables and provide useful information as regards the long-run economic relationship. If the variables have a cointegrating relationship, Markov switching vector error correction model (MS-VECM) is used to examine the variables (Wai, *et al.*, 2013). Therefore, the use of MS-VAR if there is no presence of cointegrating relationship is more appropriate.
This paper adopts Markov switching vector autoregressive (MS-VAR) approach to study the impact of inflation, import and export on economic growth in Nigeria. The remaining part of the paper is presented as follows: Section 2 is the materials and method, Section 3 presents result and discussion while Section 4 concludes the paper.

2. Materials and Method

Model Specification and description

Autoregressive Models

Autoregressive models (AR) are among the simplest time series models. Many complex forecasting models representing real phenomena are based on these models. To estimate Gross Domestic Products (GDP), we extend the AR(1) model and select the lagged of one-period economic growth, the inflation rate, the import and the export as the possible predictive variables. We first assume that all parameters of explanatory variables are subject to change at each break point. In doing this, we describe simple time-series models of the GDP where the current value is (GDP_t) based on the one-period lagged value of itself (GDP_{t-1}) and the one-period lagged value of the macroeconomic variables (X_{t-1}):

$$GDP_t = c + \lambda_1 GDP_{t-1} + \lambda_2 Inf_{t-1} + \lambda_3 Imp_{t-1} + \lambda_4 Exp_{t-1} + \varepsilon_t \quad (1)$$

where GDP_t represents the gross domestic products at time t,
Inf_t represents the Inflation rate,
Imp_t represents the import,

Exp_t denotes the export, and

ϵ_t denotes the disturbance term which is normally distributed with mean 0 and variance σ_1^2.

λ_i is parameter shift function with $i = 1, 2, 3, 4$.

Switching determined by an unobservable variable

According to Hamilton (1989), the Markov switching regime model assumes that the regime switching is determined by an unobservable state variable which follows a Markov process. This unobservable state variable is a latent variable. Statistics defines a latent variable as a variable that is not directly observed, but can be deduced from other observed variables by a mathematical model. In the Markov regime switching framework, the latent state variable St is assumed to follow a Markov process with constant transition probabilities. Under such assumption, the state variable can be estimated along with the model parameters using Hamilton filter (Hamilton, 1989). The Markov process and the Markov property are introduced below.

Markov (1906) defined a stochastic process as a Markov process if the probabilities of future values in a time series only depend on its most recent value and are independent of earlier periods, that is, the value of the current can capture all information for its prior.

$$Pr(S_t = i|S_{t-1} = j, S_{t-2} = q \cdots) = Pr(S_t = i|S_{t-1} = j) = P_{ij} \qquad (2)$$

In the Markov switching regime model, time series may change to another state, or stay in the current state at any time. The probability matrix is called the transition matrix. This study considered the transition matrix for a two state. The first order Markov chain is given as:

$$P = \begin{pmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{pmatrix} \qquad (3)$$

Where $p_{i,j=1,2}$ denotes the probability that the time series move from regime j to i. In other words, it is the probability that R_t is in the regime i conditional on which $Rt - 1$ is in the regime j. $p_{i,j=1,2}$ is,

$p_{ij} = Pr(S_t = i|S_{t-1} = j)$

Where $P_{11} + P_{12} = 1$ and $P_{21} + P_{22} = 1$.

The switching mechanism is controlled by an unobservable state variable St, and it follows a Markov process.

S_t is assumed to follow a two-state Markov process and the Markov process is assumed to be ergodic and irreducible.

The transition probabilities provide us with an expected duration that is, the length of time it takes for the system to stay in a particular regime. The expected duration is given as:

$$E(D) = \frac{1}{1 - p_{ij}}, \tag{4}$$

where $i = 1, 2$ and $i = 1, 2 \cdots$
D stands for expected duration.

Markov-switching vector autoregressive model

Krolzig (1997, 1998 and 2003) extended Hamilton's Markov switching regime models and proposed Markov switching vector autoregressive (MS-AR) models. In these extended models, the switches from one regime to another are determined by an unobservable state variable that follows a Markov process. This state variable is assumed to be common to all series in the VAR. The general form of the Markov switching regime model is:

$$\begin{gathered} R_t = c_{s_t} + \lambda_{s_t} R_{t-1} + Z_t \delta + \varepsilon_t \\ \varepsilon_t | S_t \sim N(0, \textstyle\sum_{s_t}) \end{gathered} \tag{5}$$

Where,
R_t is the vector of dependent regime variables,
S_t is the state variable, and
Z_t is the vector of independent regime variables.
The model (5) above is a 'partial' regime-dependent MS-VAR model because the parameters do not change across different regimes. If is an empty vector, then all the coefficients are subject to change and we obtain a 'pure' MS-VAR model:

$$R_t = c_{s_t} + \lambda_{s_t} R_{t-1} + \varepsilon_t \tag{6}$$

Where $R_t = (GDP_t, Inf_t, Imp_t, Exp_t)$.
The parameters can be computed using Hamilton's (1989) filter as estimation method.
The MS-VAR model offers greater flexibility in capturing potential regime shifts in a data generating process. Therefore, the need for the test of structural break in a dataset is necessary to explain the time-varying behaviour and interactions of the variables. On a general note, the characteristic of the Markov switching model is that the unobservable actualization of the regime (a period or state of economic expansion and recession) is regulated by discrete-time and discrete-state Markov stochastic process that is defined by transition probabilities

$$P_{ij} = Pr(S_{t-j} = j | S_t = i), \quad \forall_{ij} \in 1, 2, \cdots, N \tag{7}$$

St follow an irreducible ergodic N state Markov process with the transition matrix

$$P = \begin{bmatrix} p_{11} \, p_{12} \cdots p_{1N} \\ p_{21} \, p_{22} \cdots p_{2N} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ p_{i1} \, p_{i2} \cdots p_{iN} \end{bmatrix} \tag{8}$$

The assumptions of ergodicity and irreducibility are important for the theoretical properties of Markov switching vector autoregressive models (Wiri, *et al.*, 2021). A detailed discussion of the theory of Markov chains with application to Markov-switching models is given by Hamilton (1989).

Markov Switching Vector Autoregressive Model $(MS(M) - VAR(P))$

Given the general specification of an $MS(m) - VAR(p)$ model, all parameters of the autoregressive are conditioned on the state S_t of the Markov chain such that each regime $(m) - VAR(p)$ parameters follows a regime process.
Generally,

$$R_t = v_{s_t} + \sum_{i=1}^{p} b_{i,s_t} R_{t-1} + \sigma_{s_t}^{1/2} \varepsilon_i \tag{9}$$

$$R_t = \begin{cases} v_1 + \; b_{1,1} R_{t-1} + \cdots + b_{p,1} R_{t-p} + \sigma_1^{1/2} \varepsilon_i \\ . \quad . \quad . \quad . \quad . \\ . \quad . \quad . \quad . \quad . \\ . \quad . \quad . \quad . \quad . \\ v_m + b_{1,m} R_{t-1} + \cdots + b_{p,m} R_{t-p} + \sigma_m^{1/2} \varepsilon_i \end{cases} \tag{10}$$

Conditioned on the basis that $v_1 + \cdots + \sigma_1^{1/2} \varepsilon_i$ if $s_t = 1$ and $v_m + \cdots + \sigma_m^{1/2} \varepsilon_i$ if $s_t = m$ in (10).
Where,

$$R_t = \begin{pmatrix} R_1 \\ . \\ . \\ . \\ R_p \end{pmatrix}, b_i = \begin{pmatrix} b_{11} \, . \, . \, b_{1p} \\ . \; . \; . \; . \\ . \; . \; . \; . \\ . \; . \; . \; . \\ b_{p1} \, . \, . \, b_{pp} \end{pmatrix}, R_{t-i} = \begin{pmatrix} R_{t-1} \\ . \\ . \\ . \\ R_{t-p} \end{pmatrix}, s_t = \begin{cases} 1 & if s_t = 1 \\ \\ m & if s_t = m \end{cases},$$

$$v_{1,s_t} = \begin{pmatrix} v_{1,s_t} \\ . \\ . \\ . \\ v_{1,s_t} \end{pmatrix}, \quad \varepsilon_t \sim N\left(0, \sigma_{s_t}^{1/2}\right)$$

$\sigma_{s_t}^{1/2}$ is the Choleski decomposition of the return shock covariance
The b_i stands for matrix of autoregressive parameter.

3. Results and Discussion

Source of Data

Real life data on some macroeconomic variables in Nigeria such as gross domestic product (GDP), inflation rate, import and export were collected for this study. The data spanned through 1980 to 2020 from National Bureau of Statistics (NBS) and Central Bank of Nigeria (CBN) respectively. Data was converted to natural logarithms to avoid spurious regression. $\ln$gdp is the natural logarithm of Gross Domestic Product, $\ln$inf is the natural logarithm of inflation, $\ln$imp is the natural logarithm of import and $\ln$exp is the natural logarithm of export.

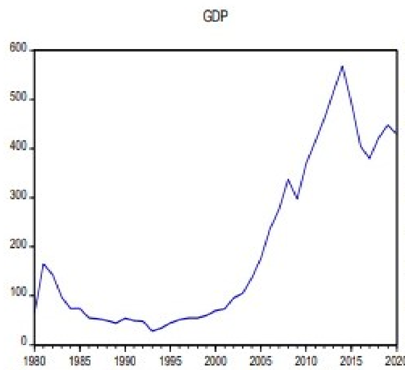

Figure 1: Time series plot of GDP

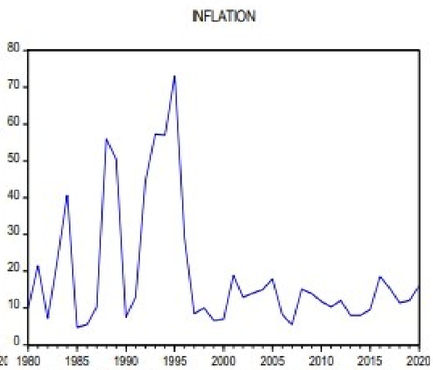

Figure 2: Time series plot of Inflation

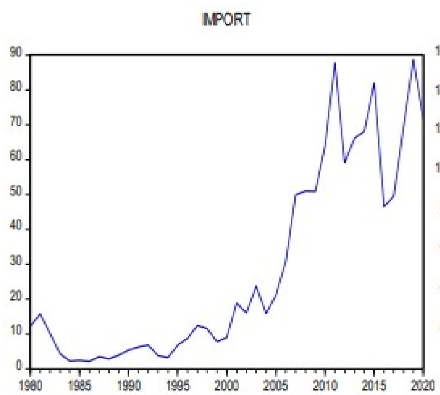

Figure 3: Time series plot of import

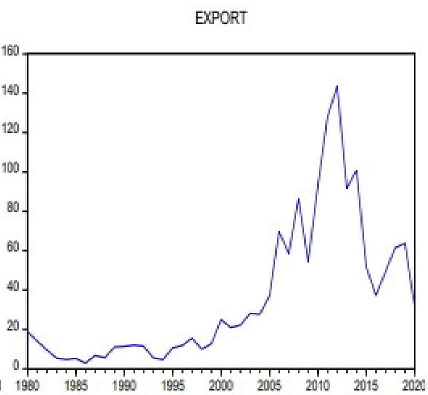

Figure 4: Time series plot of export

Figure 1 shows the time plot of GDP, GDP increased with time and was at its peak in 2013 before it falls gradually till 2020. Figure 2 shows that inflation had upward and downward movements, which indicated a regime switching structure (a cycle of high and low). In 1997, the inflation dropped drastically and moved at a lower range. Both import and export in Figures 3 and 4 had a slow upward movement until around 2005 when the rate of movement increased especially for import, indicating that goods' importation to Nigeria is helping the economy.

Table 1: Unit root test without structural breaks

Variables	Order	ADF test	ADF Inference	PP test	PP Inference
lngdp	level	-3.4593		-0.9631	
	1st difference	-3.7678**	I(1)	-44.3446**	I(1)
lninf	level	-4.4254		-21.1522	
	1st difference	-7.3375**	I(1)	-31.2870	I(0)
lnimp	level	-4.5686		-1.1458	
	1st difference	-4.2846**	I(1)	-33.221**	I(1)
lnexp	level	-2.4932		-2.9743	
	1st difference	-4.1923**	I(1)	-42.2265**	I(1)

*and ** represent 1% and 5% levels of significance respectively.

In Table 1 above, ADF and PP tests were used to test whether the variables are stationary in level or in first difference. The use of more than one unit root test was suggested by Brooks (2002) to ensure robust conclusion. The hypothesis of interest are : null hypothesis the series has a unit root against the alternative hypothesis that the series does not have a unit root (that is, stationary). All the variables were analyzed at both level and first difference, the result revealed all the variables are stationary at first difference (I(1) processes) at 5% level of significance.

Table 2: Zivot-Andrews unit root test for one structural break

Variables	t-statistics	p-value	Break date	k(lag)
lngdp	-9.3661	0.0000	1993	1
lninf	-6.7698	0.0000	1995	0
lnimp	-7.1320	0.0057	1984	1
lnexp	-7.5685	0.0001	1986	1

Critical values are -5.34, -4.80 and -4.58 for 1%, 5% and 10% level of significance respectively.

The Zivot-Andrews (ZA) unit root test result is presented in Table 2 above. The result shows that the variables are integrated of order 1 implying that the variables are stationary at first difference. The null hypothesis of a unit root is rejected at both levels of significance. The ZA break dates correspond with the transition of government as a result of coup in December 1983 that affected the economy in 1984, structural adjustment programme in 1986 by Babangida regime, election in 1993, and there was a coup in 1995.

Table 3: Lee-Strazicich unit root tests for two structural breaks

Variables	t-statistics	TB1	TB2	k(lag)	Inference
lngdp	-4.5733	1992	2009	1	NS
lninf	-7.0399	1993	1997	1	S
lnimp	-5.6869	1990	2007	2	NS
lnexp	-5.0041	1997	2011	8	NS

In Table 3 above, the results suggest that a unit root cannot be rejected for lngdp, lnimp and lnexp except for lninf. This implies that only inflation rate is statistically significant at 5% level of significance. The corresponding time of the endogenously determined structural break for inflation are 1993 and 1997. The breakpoint in 1993 occurs as a result of election that was later annulled and that of 1997 which occurred because of political crisis that leads to economic harshness in the country at that time.

Table 4: Johansen Cointegration test

No. of Relationship Vector	Trace Statistics		Maximum Eigenvalues	
	λ'_{trace}	5% C.V	λ'_{max}	5% C.V
r = 0	48.3529	47.8561	23.3394	27.5843
r = 1	25.0135	29.7971	17.7734	21.1316
r = 2	7.2400	15.4947	5.4201	14.2646
r = 3	1.8199	3.8415	1.8199	3.8415

K = number of lag = 1

There is need to test for cointegration (Johansen and Juselius(1990,1992)) despite the fact that the variables may not be from the same order of integration as Harris (1995) pointed out. This is because units roots in most cases, suffer from statistical power and size distortions problems. As we can see in Table 4 above, the trace test indicates one cointegrating relationship at 5% level while the maximum eigenvalue indicates no cointegration. It means we cannot reject the null hypothesis of no cointegration among the variables under study and this shows that there is no long run relationship among the series

Table 5: Gregory-Hansen Cointegration test with structural break

Model	Break date	ADF* test-stat.	5% C.V	k
GH1	1998	-4.93	-5.28	0
GH2	1998	-3.94	-5.57	1
GH3	1993	-5.30	-6.00	0

K = lags number by AIC

The results of the Gregory-Hansen cointegration test (Gregory and Hansen, 1996) that allows for structural breaks are presented in table 5 above. The results indicate that the ADF* test statistic is not significant at 5% significance level in all the GH models. The null hypothesis of no cointegration with structural break cannot be rejected. This implies that the series are not stationary and do not have a long run relationship. The break date for 1993 corresponds to election that moves Nigeria from military to a democratic setting but later annulled while that of year 1998 is based on regime change.

Table 6: Markov-Switching Vector Autoregressive model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Regime 1		
C	1.581853	0.095468	16.56940	0.0000
lnINF	-0.029920	0.065138	-0.459338	0.6460
lnIMP	0.609135	0.085145	7.154080	0.0000
lnEXP	-0.026123	0.094385	-0.276771	0.7820
		Regime 2		
C	1.167869	0.201907	5.784203	0.0000
lnINF	-0.039464	0.068325	-0.577602	0.5635
lnIMP	0.201858	0.150129	1.344565	0.1788
lnEXP	0.391700	0.186788	2.097023	0.0360
		Common		
$ln(\sigma)$	-2.706208	0.132200	-20.47057	0.0000
		Transition Matrix Parameters		
P11-C	2.669445	0.906097	2.946093	0.0032
P21-C	-2.293809	0.967975	-2.369697	0.0178
Mean dependent var	2.106797	S.D. dependent var		0.412930
S.E. of regression	0.120189	Sum squared resid		0.462249
Durbin-Watson stat	1.638342	Log likelihood		41.77659
Akaike info criterion	-1.501297	Schwarz criterion		-1.041558
Hannan-Quinn criter.	-1.333886			

From Table 6 above, we can deduced that both inflation and export have a negative sign on the economic growth in regime 1 while import has a positive sign implying that import contributes to Nigeria economy during the regime 1 which is statistically significant at 5% level and has a lower volatility. In regime 2, export played a significant role as it contributes immensely to the economy of the country with a higher volatility. We can also see the differences in the regime specific coefficients (Regime 1: 1.581853, Regime 2: 1.167869). Umeh and Anazoba (2016) termed such as fast and slow growth rates for Nigeria economy for the period under study. It can also be observed that regimes are significant at 5% level meaning that the dynamics of both regimes are substantial. The transition matrix parameters and log(sigma) also indicated significance in the model.

Consequently, the results in Table 7 below shows that the probability of staying in regime 1, $P(S_t = 1|S_{t-1} = 1) = 0.9352$ is higher than the probability of staying in regime 2, $P(S_t = 2|S_{t-1} = 2) = 0.9084$ which suggests that regime 1 is more persistent than regime 2. Therefore, the period of growth in regime 1 is about 16 years while the period of growth in regime 2 lasted for about 11 years. Thus, Nigeria economy has more stable state in regime 1 before movement to regime 2. This result is in line with Hamilton (1989) procedure of high and low state.

Table 7: Transition probabilities and expected duration

	1	2
1	0.935199	0.064801
2	0.091637	0.908363
$E(D_{s_t})$	15.43196	10.91262

Figure 5 below is the filtered estimates of the probabilities of being in the two regimes. Filtering is the process by which the probability estimates are updated (Tuaneh and Essi, 2021). This filtering is done to determine the likelihood of the movement from one state to the other and it shows that the states are in the years 2003 and 2005 respectively.

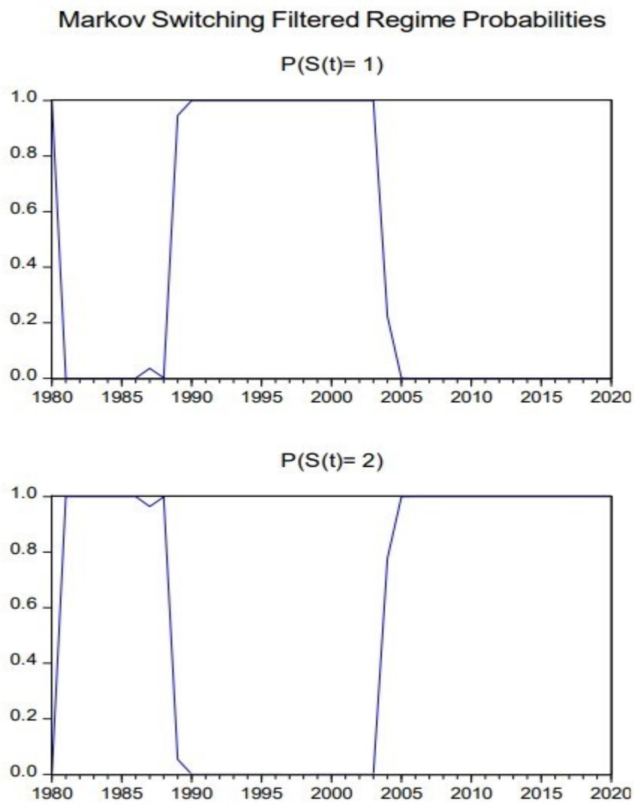

Figure 5: Chart showing the filtered estimates of the regime probabilities.

4. Conclusion

This study examined the impact of inflation, import and export on Gross Domestic product in Nigeria using Markov switching vector autoregressive model. Annual data covering 1980 to 2020 were obtained from National Bureau of Statistics (NBS) and Central Bank of Nigeria (CBN). ADF and PP unit root test without structural breaks indicted that the variables under study were stationary at first difference. Furthermore, Zivot-Andrews unit root test for one structural break indicated that variables are stationary at first difference with break years as 1984, 1986, 1993 and 1995. The Lee-Strazicich unit root test for two structural breaks was conducted but only inflation rate was stationary with break years as 1993 and 1997. The Johansen cointegration test and Gregory-Hansen cointegration test with structural breal were carried out but results shown the absence of long run relationships among the variables. Lastly, the Markov switching vector autoregressive model was applied to the macroeconomic variable, the results shows that import contributes to the Nigeria economy in regime 1 while export contributes to the Nigeria economy in regime 2. We therefor conclude that the Nigera economy has more stable state in regime 1 before movement to regime 2 implying an evidence of co-movement among the macroeconomic variables studied.

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