

## Canonical Correlation Analysis Between Plant Characteristics and Yields Component in Oil Palm (*Elaeis Guineensis* Jacq.)

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**Abstract.** Canonical correlation analysis (CCA) is a multivariate statistical technique that describes the associations between two sets of variables. In this paper a formulation of Canonical correlation analysis based on the Least Absolute Shrinkage and Selection Operator (LASSO) is utilized to examine the relationship between plant characteristics and oil palm yield. The modified CCA based on LASSO was applied to oil palm data from Nigerian Institute for Oil Palm Research (NIFOR), Nigeria. The analysis was based on an approximation using a modified optim algorithm in R statistical package. The results showed that the modified CCA approach based on LASSO distinctively selected the pairwise variables that mutually maximized the canonical correlation. The method identified optimal combinations of the key variables on oil palm characteristics for accurate prediction plant vegetative growth and development. The newly developed approach is suggested as a better alternative to the classical CCA in assessing plant development and yield.

**Keywords:** Canonical Correlation Analysis, Least Absolute Shrinkage and Selection Operator, Optim, Oil Palm

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### 1. Introduction

Canonical Correlation Analysis (CCA) has certain maximal properties that are very similar to the principal component analysis (PCA), which have been studied intensively. However, the biggest difference between these two methods is

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that CCA considers the relationship between two sets of variables, while PCA focuses on the interrelationship within only one set of variables. Moreover, CCA is more equipped in taking care of several predictors at a time by finding the pair of some basic vectors that maximize the correlation of a set of paired variables than all the multivariate technique mentioned earlier which treat only one predictor at a particular time, that is two sets of variables can be considered as different views of the same object or views of different objects, and are assumed to contain some joint information in the correlations between them.

The theory of Canonical Correlation Analysis (CCA) has developed rapidly since its introduction by Harold Hotelling in 1935. It has taken a pre-eminent position in the field of statistics and is increasingly deployed in a variety of scenarios. Several common parametric tests of significance can be treated as special cases of CCA (Knapp, 1978). The CCA technique is seen as the cornerstone of multivariate statistics, for discovery and quantification of associations between two sets of variables (Hardle and Simar, 2007). The areas of application of CCA is wide and include the sciences, engineering, agriculture, epidemiology, and the humanities.

The CCA approach explores a linear combination of all the variables in a multivariate dataset that correlates maximally with a linear composite of all the variables in the other multivariate dataset (Thompson, 1984). Muller (1982) proposed an alternative approach which presented CCA as a least squares problem. This multivariate multiple regression representation of CCA amounts to finding an estimate of  $\beta$ ,  $\alpha$  and  $D(p_k)$  in the following model equation:

$$Y\beta = X\alpha D(p_k) + \varepsilon \quad (1)$$

The matrices  $\beta$ ,  $\alpha$ , and  $D(p_k)$  correspond in the sense that the  $k^{th}$  columns of  $\beta$  and  $\alpha$  provide the linear combinations that are correlated  $\rho_k$ , which is the  $(k, k)$  element of  $D(p_k)$ . However, the multivariate formulation introduced greater complication as a result of alternating the vectors with matrices. For instance, the equivalence of  $\beta$ ,  $\alpha$ , and  $D(p_k)$  in the standard statement of CCA are vectors (not matrices). Hence, the standard statement of canonical correlation has more in common with the univariate statement than with the multivariate that was developed in the study. Hence, Vinod (1976) proposed the canonical ridge, which is an adaptation of the ridge regression concept of Hoerl and Kennard (1970) to the framework of CCA, to handle the problem of matrix inversion. The canonical ridge replaces the matrices  $C_{xx}^{-1}$  and  $C_{yy}^{-1}$  with  $((C_{xx} + k_1 I))$  and  $((C_{yy} + k_2 I))$  respectively. By adding the penalty terms  $k_1$  and  $k_2$  to the diagonal elements of the sample covariance matrices, a more reliable and stable estimates are obtained when the data are nearly or exactly collinear.

For this case, the CCA optimization problem in Hotelling (1936) is reformulated as;

$$\begin{aligned} & \max_{a_x, b_y} a_x^T C_{xy} b_y \\ \text{s.t. } & a_x^T (C_{xx} + k_1 I) a_x = 1, \\ & b_y^T (C_{yy} + k_2 I) b_y = 1 \end{aligned} \quad (2)$$

where  $k_1 > 0$ ,  $k_2 > 0$  and  $I$  is identity matrix.

However, in terms of computational time, the canonical ridge is higher com-

pared to that of CCA. Another problem with it is that the values of  $k_1$  and  $k_2$  provided by the cross validation algorithm are closely tied to the size and discretization of the search grid. Ayodeji and Obilade (2016) presented a probabilistic formulation for CCA using a novel combination of the techniques of multiple regression of two Gaussian random vectors in order to overcome the problem of inaccurate estimation associated with CCA.

In view of these limitations, this study presents a formulation of hybrid model for CCA using the techniques of least absolute shrinkage and selection operator (LASSO) regression, with implications for the simple and multiple correlations. Such formulation enhances the understanding of CCA as a model-based method that is useful in modeling and prediction. The goal is to consistently identify the variables relevant for regression and also to avoid matrix inverse computation problem usually associated with CCA, which is done for both the predictors and responses. This was used in the determination of relationship in vegetative growth in oil palm, where the estimate of the relationship between plant characteristics be independent variables and yield components be dependent variables of 210 oil palm plants population collected from Nigerian Institute For Oil Palm Research (NIFOR), Edo State, Nigeria.

Some notable application of CCA after its proposition by Hotelling (1935) was first applied in economics study by Waugh (1942) to examine the relationship between wheat characteristics to flour characteristics and hence, demonstrated that desirable wheat is high in texture, density and protein content. Thereafter, Cankaya *et al.* (2010) used CCA to estimate relationships between plant characters X set [fruit length (FL), fruit width (FW), fruit wall thickness (FWT), placenta length (PL), stem thickness (ST), plant height (PH), leaf length (LL), leaf width (LW), flowering time (FT) (50 percent), and time to maturity (TM)] as the independent variables, and yield components Y set [total fruit weight per plant (FW/P), average fruit weight (AFW) and number of fruits per plant (FN/P)] as dependent variables of 56 red peppers populations collected from the Samsun province in the Black Sea Region of Turkey. All canonical correlation coefficients (0.708, 0.635, 0.413) between the pairs of canonical variables were found to be significant ( $P < 0.01$ ). The findings obtained from the CCA indicated that FN/P had the largest contribution for the explanatory capacity of canonical variables estimated from yield components of 56 red pepper populations when compared with other yield components. Also, Balkaya *et al.* (2011) used CCA to estimate relationships between plant characteristics, X set [fruit length (FL), fruit diameter (FD), flesh thickness (FT), fiber weight per fruit (FW), length of seed cavity (LSC), skin thickness (ST), vine length (VL), branch number per plant (BN), leaf length (LL), leaf width (LW), female flowering time (50 percentage) (FFT), and time to maturity (TM)], and yield components Y set [total fruit weight per plant (FW/P), average fruit weight (AFW) and number of fruits per plant (FN/P)] of 117 winter squash (*Cucurbita maxima* Duch) in populations collected from the Black Sea Region of Turkey. The findings obtained from the CCA indicated that FW/P had the largest contribution for the explanatory capacity of canonical variables estimated from yield components of 117 Turkish winter squash (*Cucurbita maxima* Duch.) populations when compared with other yield components.

More also, Daher *et al.* (2018) used CCA to obtain estimates of coefficients of

phenotypic, genotypic, and residual correlation and evaluated the degree of association between morpho-agronomic and chemical traits in 132 interspecific hybrids between elephant grass and millet. Hence, Vieira *et al.* (2019) studied the relationship between nutrients and plant growth using the CCA. Canonical correlations showed that biomass was the most salient morphological characteristic in relation to the growth of *Tectona grandis* seedlings.

## 2. Materials and Method

### 2.1 Data Description

The study on oil palm plants population data collected from Nigerian Institute for Oil Palm Research (NIFOR), Edo State, Nigeria. We consider an  $n = 210$  observation on two sets of standardized variables  $X$  and  $Y$  in the form of the observed measurements as a multivariate response, The data matrices  $X$  and  $Y$  of sizes  $n \times p$  and  $n \times q$ , where  $n = 210$ ,  $p = 4$  and  $q = 3$ , respectively. The data is standardized such that every variable has zero mean and unit variance. This formulation will be used in the determination of relationship between plant characteristics been independent variables [ $X$  set; Fruit Height (FH), Fruit Diameter (FD), Fruit Leaflet (FL), Fruit Leaf Area (FLA)] and yield components been dependent variables [ $Y$  set; Average Fruit Weight (AFW), Number of Fruit Per Plant (NFPP), Fresh Fruit Bunch Weight (FFBW)] of 210 oil palm plants population collected from Nigerian Institute for Oil Palm Research (NIFOR), Edo State, Nigeria.

### 2.2 Canonical Correlation Analysis (CCA)

CCA model formulations by Hotelling (1935) applied the technique to a data set in which one set of variables consisted of mental tests and the other involved physical measurements. With the objective to finding pairs of some basic vectors that maximize the correlation of a set of paired variables. This was achieved by considering two set of multivariate random variables  $X \in \mathbb{R}^p$  and  $Y \in \mathbb{R}^q$ , where  $X = (x_1, x_2, \dots, x_p)$  and  $Y = (y_1, y_2, \dots, y_q)$  are column vectors (Bach and Jordan (2005)).

The CCA procedure is to choose directions  $a_x \in \mathbb{R}^p$  and  $b_y \in \mathbb{R}^q$  onto vectors  $X$  and  $Y$  respectively, such that the projections are maximize. Let  $u_x \in \mathbb{R}^n$  and  $v_y \in \mathbb{R}^n$  be projections of  $X \in \mathbb{R}^{n \times p}$  and  $Y \in \mathbb{R}^{n \times q}$ , where  $p, q \leq n$ ,  $n \leq N$ . Since CCA is based on linear transformations, we have:

$$\begin{aligned} u_x &= a_X^T X \\ v_y &= b_Y^T Y \end{aligned} \tag{3}$$

and

$C_{xx} \in \mathbb{R}^{p \times p}$  is the covariance matrix of set  $X$

$C_{yy} \in \mathbb{R}^{q \times q}$  is the covariance matrix of set  $Y$

$C_{xy} = C_{yx}^T \in \mathbb{R}^{p \times q}$  is the cross-covariance matrix between set  $X$  and  $Y$ .

The correlation coefficient  $\rho$  of  $u_x$  and  $v_y$  is given as;

$$\rho(u, v) = \frac{cov(u, v)}{\sqrt{var(u)var(v)}} = \frac{a_x^T C_{xy} b_y}{\sqrt{(a_x^T C_{xx} a_x)(b_y^T C_{yy} b_y)}} \quad (4)$$

The joint covariance matrix is then

$$\begin{pmatrix} C_{XX} & C_{XY} \\ C_{XY} & C_{YY} \end{pmatrix}$$

Now the eigenvalues and eigenvectors of the characteristic equation are computed

$$(C_{yy}^{-1} C_{yx} C_{xx}^{-1} C_{xy} - \rho^2 1) b_y = 0 \quad (5)$$

In linear CCA, the canonical correlations equal to the square roots of the eigenvalues.

Since its proposition. Here, we make use of the interventions in the area of CCA modelling, which has been explored in various fields of science (Ayodeji and Obilade (2016)). In line with the conditions of CCA, we wish to find a pair of vectors  $\alpha$  and  $\beta$  which yields set of composite variables  $X\alpha$  and  $Y\beta$  such that  $Y\beta$  is most predictable from the variables of  $X$ . In other words, we seek a pair of vectors  $(\alpha, \beta)$  that maximizes the corresponding lagrange function for equation (2) given as;

$$Q = -\frac{n}{2} \log 2\pi - \frac{1}{2} (Y\beta - X\alpha)' (Y\beta - X\alpha) - \frac{n}{2} \lambda_1 \sum_{i=1}^p |\alpha_i| - \frac{n}{2} \lambda_2 \sum_{j=1}^q |\beta_j| - \frac{K}{2} (\alpha' X' X \alpha - n) - \frac{L}{2} (\beta' Y' Y \beta - n) \quad (6)$$

where  $K$  and  $L$  denote the lagrange multipliers,  $\lambda_1, \lambda_2$  that control the shrinkage that is applied to parameters  $\alpha, \beta$  respectively.

The next step is for us to look for the first order condition, by taking derivatives with respect to  $\alpha$  and  $\beta$ , and equating to zero. Then the first order conditions in equation (3) are;

$$\frac{\partial Q}{\partial \alpha} = X' Y \beta - (1 + K) X' X \alpha - \frac{n}{2} \lambda_1 \text{sign}(\alpha) = 0 \quad (7)$$

and

$$\frac{\partial Q}{\partial \beta} = Y' X \alpha - (1 + L) Y' Y \beta - \frac{n}{2} \lambda_2 \text{sign}(\beta) = 0 \quad (8)$$

Dividing equations (4) and (5) by  $n$  and multiplying by  $\alpha'$  and  $\beta'$  we obtain the equations respectively as;

$$C_{xy} \beta - A C_{xx} \alpha - \frac{\lambda_1}{2} \text{sign}(\alpha) = 0 \quad (9)$$

and

$$C_{yx}\alpha - BC_{yy}\beta - \frac{\lambda_2}{2}\text{sign}(\beta) = 0 \quad (10)$$

where  $A = 1 + K$  and  $B = 1 + L$ , shows that  $A = B = \alpha^T C_{XY}\beta = \rho$ . Consequently, we have

$$C_{xy}\beta - \rho C_{xx}\alpha - \frac{\lambda_1}{2}\text{sign}(\alpha) = 0 \quad (11)$$

and

$$C_{yx}\alpha - \rho C_{yy}\beta - \frac{\lambda_2}{2}\text{sign}(\beta) = 0 \quad (12)$$

The solution of the stationary equations (8) and (9) will yield the canonical correlation  $\rho$  and canonical variates  $\alpha, \beta$  respectively. Thus, from equation (8) and (9) are the estimates of  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_p)$ . and are the estimates of  $\beta = (\beta_1, \beta_2, \dots, \beta_p)$  respectively.

### Algorithm for modified CCA

- Read in the dataset into the R interface.
- Input: X and Y be two data matrices.
- X as matrix of independent variables.
- Y as matrix of dependent variables.
- scale X variables.
- scale Y variables.
- Estimate the log likelihood. penalized Maximum likelihood function
- par: parameters to be estimated.
- center X will be standardized predictor matrix with no intercept column in the modeling function.
- center Y will be standardized response matrix in the modeling function.
- state penalty approach.
- calculate log likelihood.
- Penalized Log Likelihood Objective Function.
- Initial values. Lambda1 is the last term.
- Normalize the alpha parameters.
- Normalize the beta parameters.
- Obtain the Estimate Weight Parameter and Rho value.
- alpha.
- beta.
- Rho.

**Code****The Algorithm For The Modified CCA Procedure**

- Define:  $a, b, X_{n \times p}, Y_{n \times q}$  where  $p < q, U = a^T X, V = b^T Y$
- Input:  $X_{n \times p}, Y_{n \times q}$  where  $p < q$
- Calculate  $E(X) = \mu_x, E(Y) = \mu_y, Covar(X) = C_{xx}, Covar(Y) = C_{yy}, Covar(X, Y) = C_{xy}, a, b$  such that  $corr(U, V) = \rho = \frac{cov(X\alpha, Y\beta)}{\sqrt{var(X\alpha)var(Y\beta)}}$  is maximized
- Initialize  $\alpha, \beta$  to have  $L_2$  norm;
- Iterate the following two steps until convergence;
  - (a)  $\alpha \leftarrow \operatorname{argmax}_{\alpha} (Y\beta - X\alpha)^T (Y\beta - X\alpha)$  subject to  $(X\alpha)^T (X\alpha) = 1$ ;
  - (b)  $\beta \leftarrow \operatorname{argmax}_{\beta} (Y\beta - X\alpha)^T (Y\beta - X\alpha)$  subject to  $(Y\beta)^T (Y\beta) = 1$ .
- Obtain the correlation matrix
- Obtain the stationary equation  $(C_{xx}^{-1} C_{xy} C_{yy}^{-1} C_{yx} - \rho^2 1)\alpha = 0$
- Calculate: (i)  $\rho_i^{*2}$  which is the  $i^{th}$  eigenvalue of  $C_{xx}^{-1} C_{xy} C_{yy}^{-1} C_{yx}$  and  $\alpha_i$  which is its corresponding  $i^{th}$  eigenvector.
- Start if:  $i = 1 : p$   
 Maximize:  $covar(U, V)$   
 subject to:  $covar(U_i) = 1$  and  $covar(V_i) = 1$
- Calculate: (i)  $\rho_i^{*2}$  which is the  $i^{th}$  eigenvalue of  $C_{xx}^{\frac{1}{2}} C_{xy} C_{yy}^{\frac{1}{2}} C_{yx} C_{xx}^{\frac{1}{2}}$  and  $e_i$  which is its corresponding  $i^{th}$  eigenvector.  
 (ii)  $\rho_i^{*2}$  which is the  $i^{th}$  largest eigenvalue of  $C_{yy}^{\frac{1}{2}} C_{yx} C_{xx}^{\frac{1}{2}} C_{xy} C_{yy}^{\frac{1}{2}}$  and  $f_i$  which is its corresponding  $i^{th}$  eigenvector.  
 (iii) in calculating these  $i^{th}$  value, ensure that we find those linear combinations which uncorrelated with the preceding  $1, 2, \dots, i - 1$  number of canonical variables,  
 (iv)  $a_i = e_i C_{xx}^{\frac{1}{2}}$  and  $a_i = f_i C_{yy}^{\frac{1}{2}}$   
 (v)  $U_i a_i X$  and  $V_i b_i Y$
- END if
- REPORT:  $(\rho_i^{*2}, \rho_i^{*2}, \dots, \rho_i^{*2})$  and  $(U_i, V_i), \dots, (U_p, V_p)$
- END

**3. Results and Discussion**

Descriptive statistics for the examined characters are presented below;

Table 1: Descriptive values for the examined characters ( $n = 210, p = 4, q = 3$ )

Plant characters	X variable set mean $\pm$ SD	Yield components	Y variable set mean $\pm$ SD
Height	307.9 $\pm$ 86.4	Fruit Weight	2.68 $\pm$ 0.71
Diameter	153.6 $\pm$ 66.2	Number of Fruit Per Plant	3.16 $\pm$ 0.92
Leaflet	30.4 $\pm$ 6.5	Fresh Fruit Bunch Weight	1.09 $\pm$ 0.41
Leaf Area	96.2 $\pm$ 26.7		

where SD: Standard deviation.

Table 2: Bivariate correlation for variables set X and Y

	H	D	N L	AvgL	LA	B W	Avg W
Height	1.000						
Diameter	0.942**	1.000					
Number Leaflet	0.838	0.860**	1.000				
Avg Leaflets	-0.366	-0.275	-0.237	1.000			
Leaf Area	0.695	0.705	0.626	-0.115*	1.000		
Bunch Weight	0.878**	0.835	0.740	-0.307	0.617	1.000	
Avg Weight	0.667	0.653	0.526	-0.235	0.522	0.694	1.000

Table 1 shows the bivariate correlations displaying the relationships among the traits of oil palm populations. While Table 2 revealed highest correlation was predicted between paired variables, while the lowest correlations were between for all variables traits.

### 3.1 Data Application Results

Table 3: Canonical correlation of the various models on Oil Palm Data set

Model	Variable	Technique	Rho value
<b>CCA (1935)</b>	Oil Palm data	n=210,p=4, q=3 $\lambda_1 = 0, \lambda_2 = 0$	0.904
<b>Ayodeji and Obilade (2016).</b>	Oil Palm data	n=210,p=4, q=3 $\lambda_1 = 0, \lambda_2 = 0$	0.904
<b>Witten et al., (2009). PMA</b>	Oil Palm data	n=210,p=4, q=3 k=3 $\lambda_1 = 0.3, \lambda_2 = 0.3$	0.878
<b>Proposed model</b>	Oil Palm data	n=210,p=4, q=3 t= 0.1 $\lambda_1 = 0, \lambda_2 = 0$	0.904

Table 3 show the result for canonical correlation of the various models on oil palm data set from NIFOR, where it was observed that the modified CCA based on LASSO developed gave equivalent results in Rho value (P-value)when compared to others existing models.

### 3.2 Test of Significance of the Canonical Correlation Coefficient

The canonical correlation coefficients test for the existence of overall relationship between two sets of variables and redundancy measures the magnitude of relationships.

Table 4: Effect .. Within Cells Regression Multivariate Tests of Significance

Test Name	Value	Approximate F	Hypothesis DF	Error DF	Significance of F (p)
Pillais	.88535	21.45719	12.00	615.00	; 0.001
Hotellings	4.55799	76.59961	12.00	605.00	; 0.001
Wilks	.17010	42.69073	12.00	537.38	; 0.001
Roys	.81778				



In Table 4 above, the output start with a sample description and then shows the general fit of the model reporting Pillai's, Hotelling's Wilk's and Roy's multivariate criteria. The interpretation was based on Wilks's lambda ( $\lambda$ ), as it tends to have the most general applicability. The results showed that the full model was statistically significant, with a Wilks's  $\lambda$  value of 0.1701,  $F = 42.691$ ,  $p < .001$ . Following the results presented, the null hypothesis that there is no relationship between the variable sets is rejected (i.e., reject  $H_0 = 0$ ) and thus, conclude that there probably was a relationship between observed variables.

### 3.3 Hierarchical statistical significance tests in which only the last canonical function is tested

Table 5: Dimension Reduction Analysis

Roots	Wilks Lambda	F	Hypothesis DF	Error DF	Significance of F
1 to 3	.17010	42.69073	12.00	537.38	.000
2 to 3	.93345	2.38243	6.00	408.00	.028
3 to 3	.97713	2.39940	2.00	205.00	.093

The tests of dimensionality for the canonical correlation analysis as shown in Table 5, also reveals the tests of significance of each of the roots. It is seen that of the three possible roots only the first root is significant with  $p \leq .05$  while roots 2 to 3 and 3 to 3 were not statistically significant in isolation. The summary of the tests of dimensionality indicated that the relationship was largely captured by the first functions in the canonical model.

The study presents a new formulation for identifying optimal combinations of the key variables on oil palm characteristics for accurate prediction of its vegetative growth and development. The findings obtained from the CCA indicate that the correlation between first pair of canonical variates, which shows that data sets are highly correlated. The result further revealed that FFBW had the largest contribution for the explanatory capacity of canonical variables estimated from yield components of 210 Oil Palm plantations when compared with other yield components. PH and PD had largest contribution for the explanatory capacity of canonical variables estimated from plant characteristics when compared with the other characteristics.

## 4. Conclusion

This paper developed a modified canonical correlation analysis approach based on the LASSO technique. The modified CCA method based on LASSO distinctively selected the pairwise variables that mutually maximized the canonical correlation using oil palm yield data from Nigerian Institute for Oil Palm Research (NIFOR), Nigeria. The results showed that plant height and plant diameter should be used with the aim of increasing oil palm yield. The newly developed approach is suggested as a viable alternative to the classical CCA in assessing plant characteristics and yield.

## References

- Ayodeji, I. O. and Obilade, T.O. (2016). A Predictive Model for Canonical Correlation Analysis with Implications for the Simple and Multiple Correlations.
- Bach, F.R., and Jordan, M.I. (2005). A probabilistic interpretation of canonical correlation analysis, Tech. Rep. 688.
- Balkaya, A., Cankaya, S., and Ozbakir, M. (2011). Use of canonical correlation analysis for determination of relationships between plant characters and yield components in winter squash (*cucurbita maxima* Duch.) populations. *Bulgarian journal of agricultural science*, 17(5), 606-614.
- Cankaya, S. and Kayaalp, G.T. (2010). Estimation of relationship between live weights and some body measurements in German farm  $\times$  hair crossbred by canonical correlation analysis. *J Anim Prod*, 48(2), 27-32.
- Daher, R. F., Pereira, A.V., Menezes, B.R., Cassaro, S., Cecon, A.A., Furlani, E. P., Júnior, A.T., Pereira, M.G., Stida, W.F., and Vidal, A.F. (2018). Canonical correlations among morpho-agronomic and chemical traits in hybrids between elephant grass and millet. *Australian Journal of Crop Science*, 12(02), 210-216.
- Hardle, W. and Simar, L. (2007). Canonical Correlation Analysis. *Applied Multivariate Statistical Analysis*, 321-330.
- Hoerl, A.E. and Kennard, R.W. (1970). Ridge regression: Biased estimation for nonorthogonal problems. *Technometrics*, 12(1), 55-67.
- Hotelling, H. (1935). The most predictable criterion. *Journal of educational Psychology*, 26(2), 139.
- Hotelling, H. (1936). Relations between two sets of variates. *Biometrika*, 28(3/4), 321-377.
- Knapp, T. R. (1978). Canonical correlation analysis: A general parametric significance-testing system. *Psychological Bulletin*. 85(2), 410-416.
- Muller, K. (1982). Understanding Canonical Correlation Analysis through the General Linear Model and Principal Components, *The American Statistician*, 36(4), 342-346.
- Thompson, B. (1984). *Canonical Correlation Analysis: Uses and Interpretation*, Sage, Newbury Park. DOI: <https://doi.org/10.4135/9781412983570>.
- Vieira, C. R., Oscarlina, L. W., Kuang, H., and Fernando, S. (2019). Canonical Correlation Analysis Between Growth and Nutrition in Teak Seedlings. *Floresta e Ambiente*, 26(2), 2179-8087.
- Vinod, H. D. (1976). Canonical ridge and econometrics of joint production. *Journal of Econometrics*, 4(2), 147-166.
- Waugh, F.V. (1942). Regressions between sets of variables. *Econometrica, Journal of the Econometric Society*, 290-310.