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On Econometric Approach to Modeling Economic Growth

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Abstract. In this study, an econometric model was developed based on economic growth variables (EGV) and macroeconomic variables (MEV) of Nigeria using four (4) development indicators.The indicators are gross domestic product, GDP (current US Dollar), inflation rate (proxy by consumer price index, CPI), interest rate, INR $(\%)$ and exchange rate, EXR (Naira per USD). Data were collected from 1970 to 2016. The variance maximum rotation method in principal component analysis was employed. The results of the analysis aided in the classification of the variables appropriately according to the variable classification scheme at the beginning of this study. GDP and CPI are classified to positively affect economic growth variables, which means they can be used to measure Nigeria's economic growth, while interest rate and exchange rate are classified as having positive effect on macroeconomic variables for policy making.

Keywords: Economic growth, factor analysis, factor extraction, multivariate time series, principal component method.

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1. Introduction

Nigerian economy has passed through several challenges over the years. In spite of many, and frequently changing, fiscal, monetary and other macro-economic policies. Nigeria has not been able to harness her economic potentials for rapid economic development (Ogbole, 2010). Adeoye (2006) stated that the debate on the effectiveness of fiscal policy as a tool for promoting growth and development remains inconclusive. Fiscal and monetary policies are inextricably linked in macro-economic management; developments in one sector is directly proportional to the development in the other sectors. Undoubtedly, fiscal policy is central to the health of any economy, as government's power to tax and to spend affects the disposable income of citizens and corporations, and other businesses (Aregbeyen, 2007).

The relative impact of fiscal and monetary policies has been studied extensively in the literature. Friedman and Meiselman (1963), Chowdhury et al. (1986), Shapiro and Watson (1988), Chowdhury (1988), Blanchard and Quah (1989), Chari et al. (1991), Cardia (1991), Clarida and Gali (1994), Ansari (1996), Chari and Kehoe (1998), Weeks (1999), Reynolds (2000), Schmitt-Grohe and Uribe (2001) and Feldstein (2002) have examined the impact of fiscal and monetary policies on various aggregates, using various statistical models.

However, the bulk of theoretical and empirical research has not reached a conclusion concerning the relative power of fiscal and monetary policies to affect economic growth based on the models employed. Some researchers find support for the monetarist view, which suggests that monetary policy generally has a greater impact on economic growth and dominates fiscal policy in terms of its impact on investment and growth (see Ajayi (1974), Elliot (1975), Batten and Hafer (1983)], while others argue that fiscal policy stimulant are crucial for economic growth (Chowdhury et al. (1986), Olaloye and Ikhide (1995)).

However, Cardia (1991) found that monetary policy and fiscal policy play only a small role in varying investment, consumption, and output. Montiel (1989) applied a five-variable VAR model (money, wages, exchange rate, income and prices) and examined sources of inflationary shocks in Argentina, Brazil and Israel. The findings indicated that exchange rate movements among other factors significantly explained inflation in the three countries. Other studies, which have reached similar conclusions are Kamin (1996) for United states, Odedokun (1996) for Sub-Saharan Africa, Elbadawl (1990) for Uganda, Nnanna (2002) for

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Nigeria and Lu and Zhang (2003) for China. Suleman, et al (2009) in their study of money supply (M2), government expenditure, output and prices in Pakistan found that government expenditure and inflation are negatively related to economic growth in the long run while M2 positively, impact on economic growth.

Rodriguez and Diaz (1995) estimated a six-variable vector autoregressive (VAR), output growth, real wage growth, exchange rate depreciation, inflation, monetary growth, and the Solow residuals, in an attempt to decompose the movements of Peruvian output and observed that output growth could mainly be explained by 'own' shocks but was negatively affected by increases in exchange rate. Rogers and Wang (1995) obtained similar results for Mexico using a five variable VAR model. Olubusoye and Oyaromade (2008) analyzed the source of fluctuations in inflation in Nigeria using the frame work of error correction mechanism and found the lagged consumer price index (CPI) among other variables to propagate the dynamics of inflationary process in Nigeria. Omoke and Ugwuanyi (2010) in their long-run study of money, price and output in Nigeria found no cointegration vector but found that money supply granger causes both output and inflation suggesting that monetary stability can contribute towards price stability.

Onwukwe and Nwafor (2014) utilized secondary quarterly data from 1981 to 2010, and employed the newly developed multivariate time series estimation technique via Vector Autoregressive modeling to model the economic indicators in Nigeria. Gilber and Meijer (2005) proposed the time series factor analysis (TSFA) estimation methodology, which provided a way to obtain new measures that are more robust to the effects of financial innovations.

In this work, the factor analytic approach and the multivariate time series techniques were adopted to model Nigeria monetary policy. The aim of this work is to describe the covariance relationship for some selected economic variables measured over time in terms of a relatively few underlying factors, which are unobservable random quantities. The study is divided into four sections. Section one is the introduction, Section two comprises the materials and method. Section three contains the results and discussion, and the concluding remarks are given in section four.

2. Materials and method

2.1 *Description of data*

The data set used for this analysis is the annual series of the selected relevant macroeconomic variables from 1970 to 2016, spanning 47 years. The data were obtained from CBN Statistical Bulletin, 2017 and World Bank Database, 2017. Data are collected on interest rate, INR (%) and exchange rate, EXR (Local Currency Unit (LCU) per United State Dollar (USD), period average), gross domestic product, GDP (current USD), and inflation rate, CPI (proxy by consumer price index, annual %). These four variables of interest, INR, EXR, GDP and CPI are observable and can be measured.

2.2 *Factor analytic (FA) model*

Suppose we have some factors $F_1, F_2, \cdots, F_m$, which are not observable but are linearly related to a number of independent variables of interest $Y_1, Y_2, \cdots, Y_k$. Evidence for these factors is sought in the recorded values from each of the k different variables spanning T years. Each of the k variables share some common pair of values for the m factors and is a combination of those m factors. Let y_{it} be the observed variable i in the time period t. Tsay (2005) gave a general form for the time series factor analytic model as

$$y_{it} = \beta_{i0} + \beta_{ij} f_{jt} + \cdots + \beta_{km} f_{mT} + \epsilon_{km}, m < k, t = 1, \ldots, T; i = 1, \ldots, k; j = 1, \ldots, m \tag{1}$$

where β_{i0} is an intercept vector, $\{f_{ij} \mid j = 1, \cdots, m\}$ are m common factors, which should be substantially smaller than k, β_{ij} is the factor loading for variable i on the jth factor, and ϵ_{it} it is the specific factor of variable i at time t. (Gilbert and Meijer, 2005). Note that $\beta_{i0} = 0$, if the observed series are centered. The β_s are time independent but y and f are time dependent.

The data are k observable variables of interest, collected over time T.

t	Y_1	Y_2	...	Y_k
1	y_{11}	y_{21}	...	y_{k1}
2	y_{12}	y_{22}	...	y_{k2}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
T	y_{1T}	y_{2T}	...	y_{kT}

Figure 1: Data structure

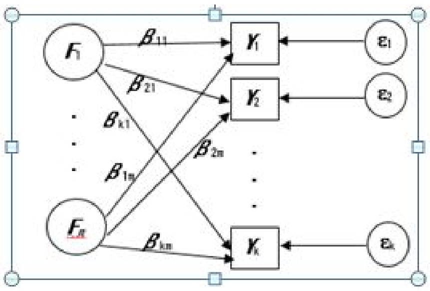


Figure 2: Structure of the model

Underlying assumptions

The factor $f_t = (f_{it}, \cdots, f_{mt})^{'}$ is assumed to be an m dimensional stationary process and are independent of ϵ_{it} such that

$$E(f_t) = \mu_f$$
$$cov(f_t) = \Sigma_f, \text{ an } m\times\ m \text{ matrix}$$

and the asset specific factor ϵ_{it} is a white noise series and uncorrelated with the common factors f_{jt} and other specific factors. Specifically, it is assumed that

$$E(\epsilon_{it}) = 0, \forall\ i \text{ and } t$$
$$cov(f_{jt}\epsilon_{it}) = 0, \forall\ i,\ j \text{ and } t$$

$$cov(\epsilon_{jt}\epsilon_{it}) = \begin{cases} \sigma_i^2, & \text{if } i = j. \\ 0, & \text{otherwise.} \end{cases}$$

Thus, the common factors are uncorrelated with the specific factors, and the specific factors are uncorrelated among each other Figure 2 is a schema designed to show the model structure. The Ys are only related to each other through their common relationship with the Fs.

In matrix form, the factor model in (1) can be written as

$$y_{it} = \beta_i^{'} F_t + \epsilon_{it} \tag{2}$$

where $\beta_i^{'} = (\beta_{i1}, \cdots, \beta_{im})^{'}$,$F_t = f_{jt}$ and the joint model for the k variables at time t is given in (3) as

$$y_t = \beta F_t + \epsilon_t t = 1, \ldots, T \tag{3}$$

where $y_t = (y_{1t}, \cdots, y_{kt})^{'}$, $\beta = [\beta_{i1}]$ is a $k \times m$ factor-loading matrix, and $\epsilon_t = (\epsilon_{1t}, \cdots, \epsilon_{kt})^{'}$ is the error vector with $E(\epsilon_t) = 0$ and $cov(f_t\epsilon_{it}) = D = diag\left(\sigma_k^2, \cdots, \sigma_i^2\right)$ a $k \times k$ diagonal matrix.

The covariance matrix of equation (3) is then given by

$$cov\left(f_t\right) = \beta\Sigma_f\beta^{'} + D \tag{4}$$

2.3 *Estimation of factor analytic model*

Let us assume the m unobserved processes of interest (the factors) for a sample of T time periods will be indicated by F_{it},$t = 1, \dots, T; i = 1, ..., m$. The k observed processes (the variables) will be denoted by y_{it}, $t = 1, ..., T; i = 1, ..., k$. The factors and variables for period t are collected in the (column) vectors F_t and y_t, respectively. It is assumed there is a measurement model relating the variables to the factors given by (1). Equation (1) is a standard FA model as earlier mentioned, except that variables are indexed by time (t) and intercepts are explicitly included, whereas in FA means are usually subtracted. A factor analysis model for the difference data allows for a linear deterministic trend in the mean of the original data as well as for a stochastic trend.

From the underlying assumptions in section 2.2, it follows that

$$p \lim_{T\to\infty} \overline{D_y} = \mu \equiv \tau + B_K \tag{5}$$

and

$$p \lim_{T\to\infty} SD_y = \Sigma \equiv B\Phi B\prime + \Omega \tag{6}$$

where D is the difference operator and SD is stationary difference operator. Conventional FA estimators use the sample covariance (or correlation) to estimate the loadings B, the factor covariance Φ, and the error covariance Ω. From (6), it follows that these estimators must also be consistent when SD_y is used as the sample covariance. Neither normality nor serial independence is required for this result. However, just as in standard FA, consistency is only obtained if B, Φ, and Ω are identified from this equation (that is, they are uniquely determined if Σ is known). Therefore, it is assumed that this is the case. Ω is assumed to be diagonal, then, if the Ledermann bound

$$(k - m)^2 \geq k + m$$

is satisfied, Ω is generally identified (Wansbeek and Meijer, 2000). As in standard FA, the parameter matrices B and Φ are uniquely defined either by imposing restrictions on their elements or by choosing a rotation method [see, e.g., Browne (2001); Loehlin, (1987)]. Given estimators $\hat{B}$, $\hat{\Phi}$, and $\hat{\Omega}$, estimators for τ and/or k can be obtained from (5).

The number of sample means in this equation is smaller than the number of parameters and therefore some restrictions must be imposed. In a typical FA model, the intercepts are free parameters, so that the means of the factors can be arbitrarily but conveniently restricted to zero, giving the restriction $k = 0$ and estimator $\hat{\tau} = \overline{D_y}$. This illustrates why the means are Ω usually neglected in FA applications. When $\tau = 0$ and k is not zero, a natural and consistent estimator of k is the GLS estimator

$$\hat{k} = (\hat{B}'\hat{\Omega}^{-1}\hat{B})^{-1}\hat{B}'\hat{\Omega}^{-1}\overline{D_y}$$

It is also possible to estimate all parameters jointly from the mean and covariance structure, i.e., use (5) and (6) jointly. Some experimentation with this did not lead to improved estimators and attention is restricted to a standard covariance-based estimator of free parameters in B, Φ, and Ω. In particular, the maximum likelihood estimator is found by minimizing

$$L \equiv log|\Sigma| + tr(\Sigma^{-1}SD^y) \tag{7}$$

where Σ is a function of the parameters, as given in (6). Resulting consistent estimators will not be full maximum likelihood, but quasi maximum likelihood. This is because the data are typically not normally distributed, may be serially dependent since the data are collected over time, and (5) may give additional information on the parameters (e.g., if $\tau = 0$), which is unused in the estimation.

Under weak assumptions, the central limit theorem implies that the elements of the sample covariance S_{Dy} are jointly asymptotically normally distributed. Let S_{Dy} be the vector consisting of all unique (non duplicated) elements of S_{Dy}, and let σ_0 be its probability limit. Then

$$\sqrt{T}(S_{Dy} - \sigma_0) \to N(0, Y_0) \tag{8}$$

for some finite positive definite matrix Y_0 (Wansbeek and Meijer, 2000). So, Y_0 can be estimated consistently by a heteroskedasticity and autocorrelation consistent (HAC) covariance estimator, such as the Newey-West estimator (Whitney, Newey and West, 1987). Stack B, Φ and Ω in the parameter vector θ and denote the population value θ_0. The estimator $\hat{\theta}$ of θ is a function of S_{Dy}. So, replacing S_{Dy} with $\hat{\theta}$ and σ_0 with θ_0, the expression in (8) gives

$$\sqrt{T}(\hat{\theta} - \theta_0) \rightarrow N(0, J_0 Y_0 J_0^{'}), \tag{9}$$

$$\hat{\theta} = g(S_{Dy}^{'})$$

$$J_0 = p \lim_{T\rightarrow\infty} [\frac{\partial\hat{\theta}}{\partial S_{_{Dy}}}]$$

The normal approximation in (9) is an implicit presentation, where $J_0 Y_0 J_0^{'}$ is the variance of $\sqrt{T}(\hat{\theta} - \theta_0)$ in distribution and its mean is zero. See Shapiro (1983) for formulas on $\frac{\partial\hat{\theta}}{\partial S_{_{Dy}}}$ for the case in which identification is obtained by explicit restrictions on the parameters.

Archer and Jennrich (1973); and Jennrich (1973) derived formulas from (9) for the case in which a rotation method is used to obtain uniquely defined parameters. Standard errors of the parameter estimators are now straightforwardly obtained and Wald and LM tests can be routinely applied if desired.

2.4 *Model specification*

Recall the time series factor analytic from equation (1)

$$y_{it} = \beta_{i0} + \beta_{i1} f_{1t} + \cdots + \beta_{k2} f_{2t} + \epsilon_{it}, t = 1, 2, \cdots, T = 47; j = 1, 2, \cdots, m = 2; i = 1, 2, \cdots, k = 4$$

The model specified for this work is the multiple time series factor analytic model given as

$$GDP_t = \beta_{GDP,0} + \beta_{GCP,1} F_{1t} + \beta_{GCP,2} F_{2t} + \cdots + \beta_{GCP,m} F_{mt} + \epsilon_{GDP,t} \tag{10}$$

$$CPI_t = \beta_{CPI,0} + \beta_{CPI,1} F_{1t} + \beta_{CPI,2} F_{2t} + \cdots + \beta_{GCP,m} F_{mt} + \epsilon_{CPI,t} \tag{11}$$

$$INR_t = \beta_{INR,0} + \beta_{INR,1} F_{1t} + \beta_{INR,2} F_{2t} + \cdots + \beta_{GCP,m} F_{mt} + \epsilon_{INR,t} \tag{12}$$

$$EXR_t = \beta_{EXR,0} + \beta_{EXR,1} F_{1t} + \beta_{EXR,2} F_{2t} + \cdots + \beta_{GCP,m} F_{mt} + \epsilon_{EXR,t} \tag{13}$$

where $F_1, F_2, \cdots, F_m$ are m factor, such that, $m < 4$, GDP = Gross Domestic Product, CPI = inflation rate, INR = interest rate, EXR = exchange rate. There are m factors and $k = 4$ variables, and $T = 47 years$. The error terms $\epsilon_{GDP,t}$, $\epsilon_{CPI,t}$, $\epsilon_{INR,t}$ and $\epsilon_{EXR,t}$ serve to indicate that the hypothesized relationships are not exact. In the special vocabulary of factor analysis, the parametersβ_{ij} are referred to as loadings. The β_{i0} for each i can be set to zero, if the observed series are centered. It should be noted that data cannot be collected on the factors because they cannot be measured or observed, data were only collected for variables (GDP, CPI, INR, EXR). Equations (10), (11), (12) and (13) show the relationship between each observed variable and the factors. Considering the Ledermann bound

$$(k - m)^2 \geq k + m \tag{14}$$

In this research, $k = 4$. This implies that, $m < 4$. So, m can be 1, 2 or 3. If $m = 3$, then the inequality in (14) will not hold. Also, if $m = 1$, then (14) will not also hold. The only possibility is that $m = 2$. Since $m = 2$, it is expected that two factors will be considered. Let assume these four variables, GDP, CPI, INR and EXR are functions of two underlying factors, F_1 and F_2. It is assumed that each of the four variables is linearly related to the two factors, say macroeconomic variable (MEV) and Economic growth variable (EGV). The first, the communality of the variable, is the part that is explained by the common factors F_1 and F_2. The second, the specific variance, is the part of the variance of Y_i that is not accounted by the common factors. If the two factors were perfect predictors of the observed variables of interest, then $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0$ always, and $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0$. To calculate the covariance of any two observable variables, say, Y_q and Y_l, we can write equations (15) and (16) as

$$Y_q = \beta_{q0} + \beta_{q1}F_1 + \beta_{q1}F_2 + \epsilon_q \tag{15}$$

$$Y_l = \beta_{l0} + \beta_{l1}F_1 + \beta_{l1}F_2 + \epsilon_l \tag{16}$$

Hence, the covariance of Y_q and Y_l is given by (17).

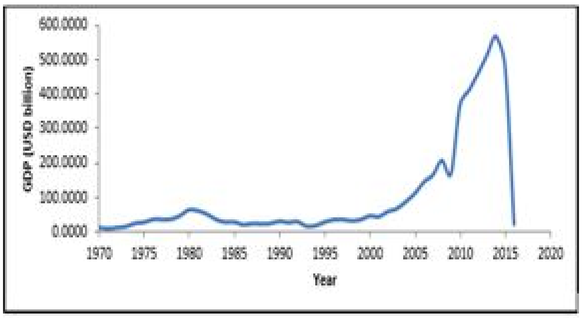
$$Cov(Y_q, Y_l) = \beta_{q1}\beta_{l1}Var(F_1) + \beta_{q2}\beta_{l2}Var(F_2) + (1)(0)Var(\epsilon_q) + (0)(1)Var(\epsilon_l) \tag{17}$$

Solving (17) further gives (18).

$$Cov(Y_q, Y_l) = \beta_{q1}\beta_{l1}Var(F_1) + \beta_{q2}\beta_{l2}Var(F_2) \tag{18}$$

3. Results and discussion

Exploratory data analysis (EDA) was carried out first before the factor analysis to expose some hidden features in the data set. The data collected from World Bank Database are updated to 2016 from the data retrieved from Central Bank of Nigeria (CBN) statistical bulletin.



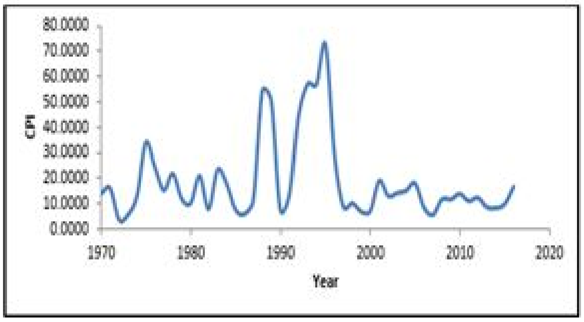
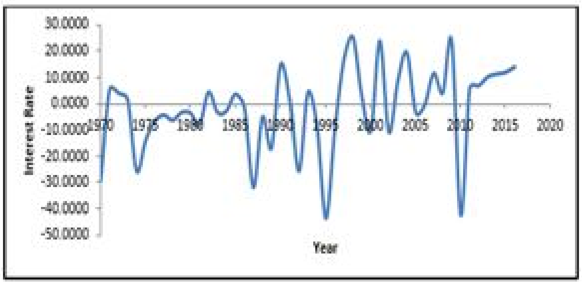


Figure 3: GDP (Current USD), 1970 -2016

Figure 4:CPI Annual %, 1970 -2016

Figure 3 shows that Nigeria economy growth rise from 1970 to 1980 but declined in 1986 but continues to rise steadily from 1999 and peaked at 2015. It dropped suddenly in 2015 to its lowest ever for the past decade. This can be term cyclical variation since the variation is not within a year. The maximum point on Figure 3 (2014) can be term economic prosperity, then the sudden decline can be term recession. As at the time this data was collected, it has not entered period of recovery. Figure 4 depicts inflation rate proxy by consumer price index (CPI), which is another measure of economic growth. The graph shows that the highest inflation rate ever recorded in Nigeria was in 1995 with inflation rate above 70%.



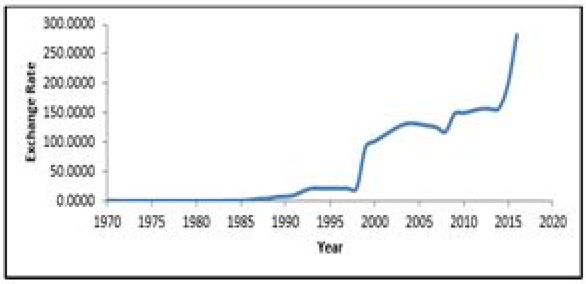
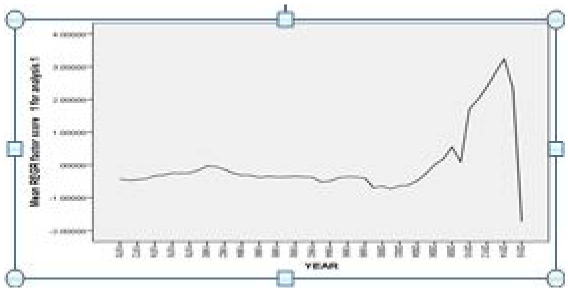


Figure 5: Interest Rate (%), 1970 -2016 Figure 6: Exchange Rate (LCU per USD), 1970 -2016

Figure 5 depicts the CBN interest rate (%) as macroeconomic variable, which is controlled by Central Bank of Nigeria (CBN). It fluctuates recording both positive and negative figures. It is highly regulated for macroeconomic policy purpose. The figure as at June 2016 was 14% . Figure 6 depicts the exchange rate, measured in local currency, naira. Exchange rate naira to United State of America dollar (USD) declined steadily from below N1.00 in 1970 to above N1.00 in 1986, which was the first time when the USD value was higher than naira, thus leading to devaluation of the naira. The figure also shows that there were gallops increment from time to time and peaked at N282.50 as at June 2016 and it is currently above N400.00 in August 2016

Table 1: Descriptive statistics

Variables	Range	Minimum	Maximum	Mean	Std. Dev.	Skewness	Kurtosis
GDP	559.326	9.182	568.508	103.057	149.289	2.095	3.169
CPI	69.378	3.458	72.836	18.620	16.054	1.905	2.964
INR	68.855	-43.573	25.282	-1.706	15.811	-0.779	0.728
EXR	281.953	0.547	282.500	59.456	72.718	0.990	0.117



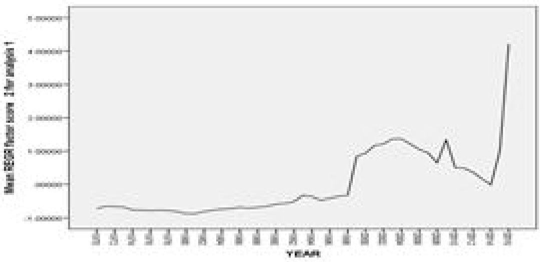


Figure 7: Factor 1 (MEV), 1970 – 2016 (June) Figure 8:Factor 2 (MEV), 1970 – 2016 (June)

Table 1 shows the descriptive analysis of four variables, which are used as cases in the factor analysis. Figure 7 and Figure 8 depict the time plot of factor 1 (EGV) and factor 2 (MEV). Factor 1, measures the economic growth of Nigeria and it shows that Nigeria economy was trending upward until 2015, when it experienced a sharp decline to 2016. The second factor, factor two measures macroeconomic variable which is used to stabilized the economy. Table 2 shows that there is a significant negative correlation between GDP and CPI. The correlation between GDP and INR is positive but is not significant, while the correlation between GDP and EXR is positively high and very significant. CPI has significant negative correlation with INR and EXR. INR has a significant positive correlation with EXR.

Table 2: Correlation matrix (p-values in bracket)

	GDP	CPI	INR	EXR
GDP	1			
CPI	-0.267 (0.035)	1		
INR	0.187 (0.104)	-0.426 (0.001)	1	
EXR	0.653 (0.000)	-0.266 (0.035)	0.348 (0.008)	1

Table 3: KMO and Bartlett's test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy		0.580
Bartlett's Test of Sphericity	Chi-Square	41.002
	D.f	6
	P-value	0.000

Table 3 shows the Kaiser-Meyer-Olkin measure of sampling adequacy based on correlation. This measure varies between 0 and 1, and values closer to 1 are better. A value of 0.58 is approximately 0.6, which is a suggested minimum. The Bartlett's test of sphericity tests the null hypothesis that the correlation matrix is an identity matrix. These tests provide a minimum standard, which should be passed before a factor analysis (or principal components analysis) should be conducted. In this case, the test is significant, we can continue with the factor analysis.

Table 4: Communalities

Communalities				
	Raw		Rescaled	
	Initial	Extraction	Initial	Extraction
GDP	22287.217	22287.163	1.000	1.000
CPI	257.737	23.613	1.000	0.092
INR	249.994	35.289	1.000	0.141
EXR	5287.964	5285.047	1.000	0.999
Extraction Method: Principal Component Analysis.				

Table 4 shows the communalities for the raw and rescaled data. The values in the extraction column indicate the proportion of each variable's variance that can be explained by the retained factors. GDP and EXR are well represented while CPI and INR are not well represented. The values in the extraction column are the reproduced variances from the factors that we have extracted. You can find these values on the diagonal of the reproduced covariance matrix displayed in Table 6. We have two factors which are

Table 5: Total variance explained

	Component	Initial Eigenvalues[a]			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
		Total	% of Variance	Cum %	Total	% of Variance	Cum %	Total	% of Variance	Cum %
Raw	1	24891.24	88.64	88.64	24891.24	88.64	88.64	21694.65	77.25	77.25
	2	2739.88	9.76	98.39	2739.88	9.76	98.39	5936.46	21.14	98.39
	3	311.55	1.11	99.50						
	4	140.25	0.50	100.00						
Rescaled	1	24891.24	88.64	88.64	1.66	41.58	41.58	1.17	29.26	29.26
	2	2739.88	9.76	98.39	0.57	14.23	55.81	1.06	26.54	55.81
	3	311.55	1.11	99.50						
	4	140.25	0.50	100.00						

EGV and MEV, so we have specified two factors for the analysis since the four variables are assumed to be classified into two categories. Thus, the two factors are retained. The eigenvalues are the variances of the factors. The first two factors together account for 98.391% of the total variance. This shows that only two factors are needed. The number of rows in this panel of the table correspond to the number of factors retained. We requested that two factors be retained, so there are two rows, one for each retained factor.

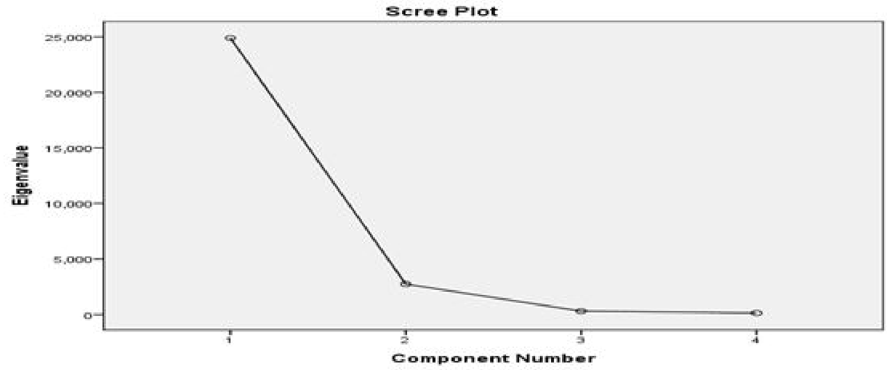


Figure 9: Scree plot

Figure 9 depicts the scree plot, which graphs the eigenvalue against the factor number. This supports the result in Table 5 that only two factors accounted for 98.391% of the total variance. Table 6 contains two tables, the reproduced covariances in the top part of the table, and the residuals in the bottom part of the table. It is expected that the values in the reproduced matrix is as close to the values in the original covariance matrix as possible. This implies that the residual matrix, which contains the differences between the original and the reproduced matrix to be close to zero. The larger the communality, the more successful

Table 6: Reproduced covariances

		GDP	CPI	INR	EXR
Reproduced Covariance	GDP	22287.163	-643.163	441.034	7093.548
	CPI	-643.163	23.613[a]	-24.313	-328.386
	INR	441.034	-24.313	35.289[a]	423.941
	EXR	7093.548	-328.386	423.941	5285.047[a]
Residual	GDP		2.871	.822	.040
	CPI	2.871		-83.902	17.424
	INR	.822	-83.902		-23.539
	EXR	.040	17.424	-23.539	

the postulated factor model can be said to be in explaining the variable.

The principal component method determines the values of the β_{ij} which make the total communality approximate as closely as possible the sum of the observed variances of the variables.

Table 7: Principal component solution (unstandardized variable)

Variable (Y_{it})	Observed Variance (S_i^2)	Loadings on F_1 $\hat{\beta}_{i1}$	Loadings on F_2 $\hat{\beta}_{i2}$	Communality $\hat{\beta}_{i1}^2 + \hat{\beta}_{i2}^2$	Percentage explained (%)
GDP (Y_1)	22287.217	148.210	-17.917	22287.163	100.000
CPI (Y_2)	257.737	-4.547	-1.715	23.613	9.162
INR (Y_3)	249.994	3.551	4.762	35.289	14.116
EXR (Y_4)	5287.964	53.775	48.921	5285.047	99.945
Overall Sum of squared loadings	28082.912	24891.240	2739.901	27631.112	98.391

The loadings on F_1 are relatively large for Y_1 and Y_4 but close to zero for Y_2 and Y_2; the loadings on F_2 are close to zero for Y_1, Y_2, andY_3 but relatively high for Y_4. Thus, F_1 could be interpreted as economic growth variable (EGV), and F_1 as macroeconomic variable (MEV). It is at this point, we have knowledge of what EGV and MEV actually are. We also observe that the factor model explains 100%, 9.2%, 14.1% and 99.0% respectively of the observed variance of GDP, CPI, INR and EXR. Overall, the two factors explain 98.39% of the sum of all observed variances. The estimate of the specific variance of Y_1, σ^2, is the differences between the observed variance and estimated communality of Y_1. It should be noted that the data used here are not measured in the same unit, which might affect the outcome of the analysis. There is need to standardized the data. Having the total communality approximate as closely as possible, the sum of the observed variances

Table 8: Estimate of specific variance

Variable (Y_{it})	Observed Variance	Estimated Communality	Estimated specific variance
GDP (Y_1)	22287.217	22287.163	0.054
CPI (Y_2)	257.737	23.613	234.124
INR (Y_3)	249.994	35.289	214.705
EXR (Y_4)	5287.964	5285.047	2.917

makes sense when the Y variables are measured in the same units. When this is not so, however, it is clear that the principal component method will favour the variables with large variances at the expense of those with small ones. For this reason, it is customary to standardize the variables prior to subjecting them to the principal component method so that all have mean zero and variance equal to one. This is accomplished by

$$Y_{ij}^t = \frac{Y_{ij} - \bar{Y}_i}{S_i}$$

whereY_{ij}each observation, $\bar{Y}_i$ is the mean of variable i, S_i standard deviation of variable i and Y_{ij}^t is the standardized observation. Table 9 shows the stability and variability of the variables used in the factor

Table 9: Estimate of variables

	GDP (Y_1)	CPI (Y_2)	INR (Y_3)	EXR (Y_4)
Mean	103.057	18.620	-1.706	59.456
Varinace	22287.217	257.737	249.994	5287.964
Std dev.	149.289	16.054	15.811	72.718

analysis. The observations of the standardized variables are depicted in Figure 8.

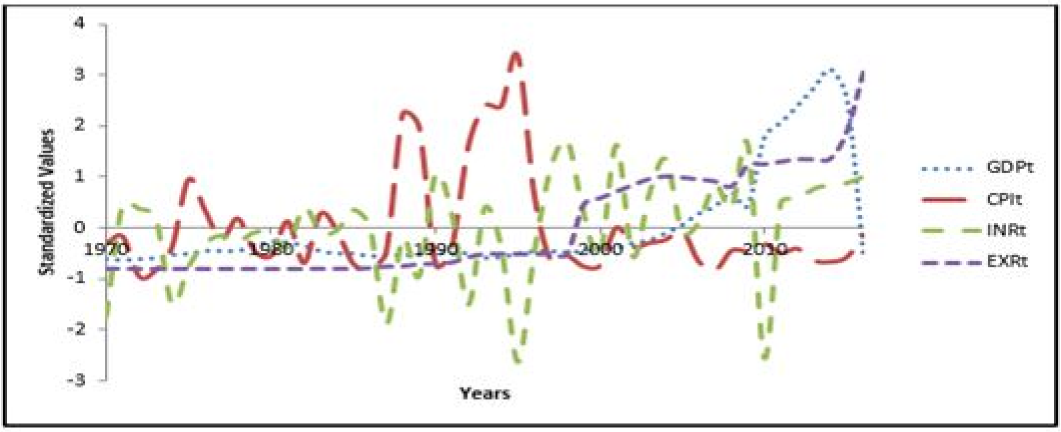


Figure 10:Time plot of standardized variable

As with the original unstandardized variables, GDP and EXR depend on one common factor (which can be interpreted as economic growth variable (EGV)) but not appreciably on the other (macroeconomic variable (MEV)); the reverse holds for CPI and INR.

Factor rotation

The varimax method encourages the detection of factors each of which is related to few variables. It discourages the detection of factors influencing all variables.

Table 10: Principal component solution computed from the standardized variable

Standardized Variable (Y_{ij}^t)	Observed Variance (S_i^{t2})	Loadings on F_1 $\hat{\beta}_1$	Loadings on F_2 $\hat{\beta}_2$	Communality $\hat{\beta}_{i1}^2 + \hat{\beta}_{i2}^2$	Percentage explained (%)
GDP (Y_1)	1	0.993	-0.12	1	100.000
CPI (Y_2)	1	-0.283	-0.107	0.092	9.200
INR (Y_3)	1	0.225	0.301	0.141	14.100
EXR (Y_4)	1	0.74	0.673	0.999	99.900
Overall Sum of squared loadings	4	1.664	0.569	2.232	55.800

Table 11: Factor transformation matrix

Factor	1	2
1	0.925	-0.380
2	-0.380	0.925

Table 11 shows the factor score covariance matrix. Since we used an orthogonal rotation, this displayed a diagonal matrix. In actuality, the factors are uncorrelated; however, because factor scores are estimated there may be slight correlations among the factor scores.

Table 12: Rotated factor matrix

	Raw Factor		Rescaled Factor	
	1	2	1	2
GDP	143.906	39.728	0.964	0.266
CPI	-3.555	-3.313	-0.221	-0.206
INR	1.476	5.754	0.093	0.364
EXR	31.160	65.682	0.429	0.903

Table 12 contains the rotated factor loadings (factor pattern matrix), the rescaled factor, which represent both how the variables are weighted for each factor but also the correlation between the variables and the factor. Since these are correlations, possible values range from -1 to +1. For orthogonal rotations, such as varimax, as used in this research, the factor pattern and factor structure matrices are the same.

The number of factors and their nature were hypothesized in advance. It was reasonable to assume that economic growth variables (EGV) and macroeconomic variables (MEV) were two factors influencing the four development indicators, that is, GDP, CPI, INR and EXR.

Table 13: Varimax rotation of standardized variables

Standardized Variable (Y_{ij}^t)	Observed Variance (S_i^{t2})	Loadings on F_1 $\hat{\beta}_1$	Loadings on F_2 $\hat{\beta}_2$	Communality $\hat{\beta}_{i1}^2 + \hat{\beta}_{i2}^2$	Percentage explained (%)
GDP (Y_1)	1	0.964	0.266	1	100.000
CPI (Y_2)	1	-0.321	-0.206	0.245	14.528
INR (Y_3)	1	0.193	0.464	0.253	25.315
EXR (Y_4)	1	0.429	0.903	0.999	99.945
Overall Sum of squared loadings	4	1.171	1.061	2.397	62.425

Table 13 shows that 100% of the variation in GDP is explained by the variation in the two factors, 99.945% of the variation in EXR is explained by the variation in the two factors; while 25.315% and 14.528% of the variation in CPI and INR are explained by the variation in the two factors. This simply means that GDP and

EXR are two variables that are well explained by the variations in MEV and EGV, while CPI and INR are not well explained by the variations in MEV and EGV.

Table 14: Component score coefficient matrix

	Component	
	1	2
GDP	1.193	-0.565
CPI	0.001	-0.010
INR	-0.008	0.026
EXR	-0.348	1.261

Table 14 shows the factor score coefficient. We can now use this score to fit the factor analytical model for multivariate time series model. The model is fitted thus:

$$GDP_{it} = 1.193EGV_t - 0.565MEV_t$$

$$CPI_{it} = 0.001EGV_t - 0.915MEV_t$$

$$INR_{it} = -0.008EGV_t + 0.026MEV_t$$

$$GDP_{it} = -0.48EGV_t + 1.261MEV_t$$

This final rotated result has actually helped to classify the variables appropriately. GDP and CPI are classified to be positively affected by economic growth variables (EGV), while interest rate and exchange rate are classified as having positive relationship with macroeconomic variables (MEV). This means that GDP and CPI can be used to measure the economic growth of Nigeria, while INR and EXR are variables for policy making, thus, are variables needed to be controlled to stabilize the economy.

4. Conclusion

The analysis shows that factor 1 is positively related to gross domestic product and consumer price index of Nigeria, while factor 2 is positively related to interest rate and exchange rate. This final rotated result has actually helped to classify the variables appropriately. GDP and CPI are classified to positively affect economic growth variables, which means they can be used to measure the economic growth of Nigeria, while interest rate and exchange rate are classified as having positive effect on macroeconomic variables for policy making, thus, are variables needed to be controlled to stabilize the economy.

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