

A Meta-Goal Programming Model with Lexicographic Achievement Function for Multi-Product Systems

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Abstract. Goal programming approach is used for solving multi-objective optimization problems that balances the trade-off between conflicting goals. The meta-goal programming framework gives the decision maker the ability to add a new set of goals which are in some sense goals of the initial goals. Meta-GP model is formulated by adopting any of the alternative goal programming variants (i.e. weighted, lexicographic or minmax etc.) in its achievement function. The particular variant used in meta-GP depends on the structure of the problem being solved. In this paper we present a meta-goal programming model with lexicographic meta-achievement function and mixed-integer variables for the achievement of management's goals in multi-product manufacturing systems. The formulated meta-goal programming model was illustrated with data collected from a multi-product company (Nigeria Breweries). With eight initial set of goals and three meta-goals, the results obtained using the LINGO optimization software showed that the meta-goal programming model with lexicographic meta-achievement function gives a more efficient solution when compared to that of a lexicographic GP model. The solution of the meta-goal programming model with lexicographic meta-achievement function was also checked for lexicographic redundancy and it was seen that the solution was not redundant. Hence the model was considered valid. A comparison of the solution of the meta-goal programming model with the lexicographic meta-achievement function with that of a meta-GP model with weighted meta-achievement function also showed that the former gives more satisfactory solution.

Keywords: Goal programming, Meta-Goal programming, lexicographic meta-achievement function, multi-product goal model, mixed-integer goal programming model.

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1. Introduction

Multiple objective optimization techniques handle problems that have several targeted goals which are generally not measurable in the same units and often conflicting. The solutions obtained in such models are often not optimal but said to be compromised based on the relative importance of each objective. The Goal programming (GP) technique is appropriate for solving such multi-objective models: see Charnes and Cooper (1975). The Goal Programming technique has been applied in a wide range of planning, resource allocation, policy analysis and functional management problems (Sharma, 2013). The major variants used in these applications of the goal programming technique with respect to distance metric includes the Lexicographic, Weighted and Minmax GP variants though a vast majority of the early goal programming formulations used the Lexicographic goal programming variant. The distinguishing feature of the Lexicographic variant is the existence of a number of priority levels with each priority level containing a number of unwanted deviations to be minimized. In the solution to a problem using either the lexicographic, weighted or Minmax GP variants the decision maker may not entirely accept the initial solution obtained. This might be due to wide variation from the different targets of the goals or as a result of a large number of unmet goals. This problem can be tackled by carrying out sensitivity analysis, which entails changing some parameters of the model with the same variant, while taken into account some feedback from the decision maker. This approach actually take a lot of time to achieve the desired result.

The meta-goal programming technique proposed by Uria, Caballero, Ruiz and Romero (2002) allows the decision maker more flexibility in expressing their preferences by means of setting meta-goals. In this Meta-

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GP approach the decision maker sets meta-goals, which can also be referred to as secondary goals, on some initial goals. The meta-goal programming model can be formulated with either the Weighted, Lexicographic or Minmax meta-achievement function as deemed appropriate by the decision maker. The weighted meta-achievement function is used when the decision maker intends to minimize the weighted sum of unwanted deviations from the set of meta-goals while the lexicographic meta-achievement function is used when the decision maker prioritizes the achievement of the meta-goals in the meta-achievement function. On the other hand the Minmax meta-achievement function is used when the decision maker intends to minimize the maximum unwanted deviations from the set of meta-goals.

The Lexicographic GP model as well as Meta-goal programming model has been applied in various areas of study by different researchers. The Meta-GP model proposed Uria et al. (2002) was used to establish target values for weighted sum of unwanted deviation variables, number of goals fully achieved as well as for maximum unwanted deviation in a hypothetical production planning problem with six goals targets : environmental impact goal, gross margin-break-even-point goal, employment goal, production capacity goal, profit goal and gross margin goal. Meanwhile two new meta-objectives were introduced into the extended goal programming framework by Jones and Jimenez (2013). The first meta-objective added was the number of unmet goals while the second was the measure of closeness to the pairwise comparisons given by the decision maker. Yazdi, Fallalipour and Moghaddam (2017) optimally selected some portfolios using the Meta-Goal Programming (MGP) and Extended lexicographic Goal programming (ELGP) models. The goal was to maximize the rate of return and minimize the risk of liquidity in the Tehran exchange market. Meanwhile a Meta-GP approach within the interactive framework was formulated by Caballero, Ruiz, Uria and Romero (2006). Lin, Nagalingam and Lin (2007) proposed an interactive Meta-goal programming model that overcomes the distributive decision making challenges that is faced by small and medium manufacturing enterprises engaged in collaborative manufacturing. Mardani, Kenari, Babaei and Asemani (2013) presented a Meta-GP approach for optimal allocation of land under cultivation and proposed a seasonal agricultural plan for different crops in the Yazd district in Iran. Acha and Iwuji (2019) presented a meta-goal programming model with weighted meta-achievement function which was applied to a multi-product manufacturing system. The unwanted meta-goals in the achievement function were weighted but without priorities on them. This implies that all the meta-goals were considered to be of equal importance but weights were attached to those meta-goals deviational variables in which we intend to minimize deviations towards them. The formulated model was illustrated with data collected in a multi-product company and the meta-GP model produced satisfying result.

From the literature available to authors, none has applied the meta-goal programming model with lexicographic achievement function to production system. This work contributes to the literature by framing a meta-goal programming model with the lexicographic option in the achievement function for multi-product systems. In this case the achievement function of the model will comprise a set of priority levels. The model presented is well suited to a multi-product system wherein a set of non-overlapping goals in a given period of time is defined.

The structure of this paper is as follow. Following this introductory part is section 2 which is the methodology where we presented meta-Goal Programming model with lexicographic structure and mixed-integer variables for multi-product systems which is followed by the assumptions of the meta-GP model. In section 3 an illustration of the Meta-Goal Programming model with lexicographic structure is presented using data collected from Nigeria breweries PLC Kaduna State production factory as used in Acha and Iwuji (2019). This is followed by result and interpretation of the result. In the last part of section 3 the result obtained with the meta-Goal Programming model with lexicographic structure is checked for lexicographic redundancy. Section 4 concludes the paper.

2. Methodology

2.1 Meta-GP model with lexicographic meta-achievement function

The mission of Nigeria Breweries PLC is to be the leading beverage company in Nigeria and so intends to market high quality brands for its customer satisfaction (NBPlc, 2019). Hence its operational goal is to produce quality drinks that are appropriately distributed to different consumption points for customers' satisfaction. Also like all business enterprises prioritize maximizing profit for shareholders satisfaction as well as reducing operational cost (Tang, 2008), the operational goals of Nigeria Breweries PLC includes improving profit for its shareholders as well as minimizing operations cost. Hence in this work we formulate

a meta-goal programming model that minimizes underachievement of estimated profit targets, minimizes underachievement of estimated production targets according to customers' demands as well as minimizes the distribution costs for its monthly production subject to some raw materials constraints. The mixed-integer meta-GP model with lexicographic structure is presented as follows.

Let the products of the company be $\mathbf{X} = (x_1, x_2, \dots, x_n)$. Given that the management of the system have q goals ($q=1,2,\dots,Q$) which they have set to achieve in a certain period of time. The goals includes minimizing deviations from some estimated set targets given as b_q , with n_q as underachievement of the targets while p_q is the overachievement of the target. Meanwhile r meta-goals ($r = 1, 2, 3$) are set by management on the initial q goals so as to capture the structure of its preferences and coherently to obtain a more satisfactory solution when the initial solution obtained is not acceptable. The goals as well as the meta-goals are categorize in priority levels, l , by management with $l=1,2,\dots,L$.

Let the meta-goals considered by management include- the meta-goal relating to the percentage sum of unwanted variables for all goals, the meta-goal relating to the maximum percentage deviation for some or all goals and the meta-goal relating to the maximum number of unachieved goals allowed. Let the maximum bound of the r meta-goals be B_1, B_2 , and B_3 . These bounds are actually target values set by management in the given period for each of the meta-goals. α_r and β_r represents the underachievement (negative deviation) and overachievement (positive deviation) respectively of the r th meta-goal. Each meta-goal is a function which is a subset of the initial goals.

Let α_r = negative deviation (underachievement) associated with the r th meta-goal. β_r = positive deviation (overachievement) associated with the r th meta-goal. u_r^l = preferential weights associated with the minimization of α_r in the l th priority level. v_r^l = preferential weights associated with the minimization of β_r in the l th priority level. k_q = normalization constant associated with the q th goal. g_j = total availability of the j th resource for a given period. n_q = negative deviational variable for the q th goal. p_q = positive deviational variable for the q th goal. Q_i = set of goals involved in the i th priority level meta-achievement function. w_q = weight associated with the q th goal. M = arbitrarily large values that the corresponding attributes cannot achieve.

Then the meta-GP model with lexicographic meta-achievement function which presents a prioritized minimum deviation policy from 3 meta-goals for a multi-product system is presented as follows:

Lexicographic Minimize

$$d = \sum_{r=1}^3 (u_r^l \alpha_r + v_r^l \beta_r), \quad l=1,2,\dots,L \quad (1)$$

subject to the constraints

$$\sum_{j=1}^m a_{qj} x_j + n_q - p_q = b_q \quad (2)$$

$$\sum_{j=1}^m c_{ij} x_j (\leq, =, \geq), \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad (3)$$

$$\sum_{q=1}^Q w_q \left[\frac{n_q}{k_q} + \frac{p_q}{k_q} \right] + \alpha_1 - \beta_1 \leq B_1, \quad \text{for some goals} \quad (4)$$

$$w_q \frac{n_q}{k_q} - \lambda \leq 0 \quad (5)$$

$$w_q \frac{p_q}{k_q} - \lambda \leq 0 \quad \text{for either } n_q \text{ or } p_q \text{ for each goal } q \quad (6)$$

$$\lambda + \alpha_2 - \beta_2 \leq B_2 \quad (7)$$

$$n_q - My_q \leq 0 \quad (8)$$

$$p_q - My_q \leq 0 \quad \text{for either } n_q \text{ or } p_q \text{ for each goal } q \quad (9)$$

$$\sum_{q=1}^Q y_q + \alpha_3 - \beta_3 \leq B_3 \quad (10)$$

$$\begin{aligned} y_q &= 1, & \text{if goal } q \text{ is not satisfied, for some/all goals} \\ &0, & \text{otherwise} \\ x_1, x_2, \dots, x_n &\geq 0 \text{ and integer,} \end{aligned} \quad (11)$$

$$n_q, p_q \geq 0 \text{ and non-integer for some } q. \lambda, \alpha_r, \beta_r \geq 0, \alpha_r \times \beta_r = 0 \quad (12)$$

$k_q = 1$ if the meta-goal is a subset of goals in different units while $k_q = 0$ when the meta-goal is a subset of goals in same units.

$\alpha_r \times \beta_r = 0$ which implies that in the meta-achievement function, the deviational variable of the meta-goal which is not desired to be minimized for the given priority level is expected to be zero.

In the model presented, eqn. (1) represents the lexicographic meta-achievement function of the model which comprises the underachievement and overachievement deviational variables (i.e α_r and β_r respectively) of meta-goals to be achieved. These deviational variables shows by how much the achieved goals' values deviated from the targets either positively or negatively. Such meta-goals are set by the company depending on managements desired objectives in a given point in time. The meta-goals are set at different priority levels according to managements' objectives. Each priority level contains the deviational variables of the goal's target to be minimized with its weight. The weights, u_r^l and v_r^l , in a given priority level l shows the relative importance of the minimization of the associated deviational variable in that priority level. In goal programming the weight of '1' is given to those deviational variables whose minimization are accepted while weights of '0' are given those whose minimization are not accepted. The meta-goals that fall in a given priority level are determined by the decision makers. Meanwhile constraint (2) represents the goals constraint with a_{qj} being the quantity of the goals like profit, production quantity and cost associated with the production of each drink while b_q is the target of each goal set to be achieved by the company. Constraint (3) is the system constraint with c_{ij} being the quantity of raw material i used in producing drink j in the production run and g_j is the total available quantity of the j th resource. Constraint (4) is for the first meta-goal (relating to percentage sum of unwanted variables for all goals). This percentage sum for the first meta-goal is an upper bound B_1 set by management for attainment of this meta-goal. Constraints (5), (6) and (7) are all constraints for the second meta-goal (meta-goal relating to the maximum percentage deviation for some/all goals), with λ as maximum deviation from all/some of the goals while B_2 is the upper bound for the second meta-goal as set by management. Constraints (8), (9), (10) and (11) are all constraints for the third meta-goal (meta-goal relating to the maximum number of unachieved goals allowed), with y_i as binary variable associated with number of unsatisfied goals allowed in the model while B_3 is the upper bound of unachieved goals set by management.

The variables of the model are mixed-integer since some variable are integer constrained while others are continuous. The deviational variables n_q and p_q are integer constrained for the goals relating to quantity of drinks produced since drinks are produced in whole and not in fractions, while, on the other hand, the deviational variables are non-integer for the goals relating to profit and cost since they can take continuous values. The decision variables are $x_1, x_2, \dots, x_n$ and they are integer constrained since the number of truckload of drinks produced is considered to be in whole and not fractions.

2.2 Assumptions of the meta-GP model

The general assumptions of the Meta-Goal Programming model are as follows;

- **Additivity:** Additivity assumption implies that the level of penalization for undesired deviational variables from a target level does not depend on the levels of unwanted deviational variables from the other goals.
- **Proportionality:** Proportionality assumption in the goal programming model requires that the penalization for an unwanted deviational variable from a target level is directly proportional to the distance away from the target level.
- **Divisibility:** This assumption implies that all the decision variables should be free to take any value within their stated range, i.e., a decision variable cannot be forced to take an integer or a discrete value.
- **Certainty:** This assumption implies that all the data coefficients are known with certainty.

3. Data analysis, results and discussion

3.1 Data analysis

The Meta-goal programming model formulated above is illustrated using data collected from Nigeria Breweries factory in Kaduna State Nigeria for the production of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export. These drinks considered are the six drinks with a very high demand produced at the Kaduna breweries. The sample size of the six drinks is arbitrarily chosen. There were some limitations in the data collection. These limitations includes the inability to obtain exact values of some parameters like profit and cost targets of the company. Though some part of the parameters, like the quantity of the raw materials used in brewing for each drink and quantity of drinks produced per month, were collected by direct interview of workers in the production unit, the profit and cost target parameter values we were not given. So the profit value was obtained by taking 20% percentage of total sales as profit while the cost of distribution was taken as the cost of fueling the trucks in the given time interval.

The goals considered are as follows;

- Minimization of the underachievement of the estimated total profit target.
- Minimization of the underachievement of the estimated monthly production target levels for each of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export.
- Minimization of the overachievement of the estimated monthly distribution cost.

The underachievement of the profit target might be caused by factors like increase government taxation and raw material cost as well as inflation which may reduce consumption from customers. The production of drinks targets may also be underachieved due to machine failure among other things while distribution cost might be overachieved due to increase in the cost of diesel used by the trucks. We have 8 initial goals. The estimated monthly profit per truckload (700 crate capacity) of each drink, as presented in Table 1, is gotten by assuming a profit level of 20% from sales value (per truckload) while the estimated monthly distribution cost (fueling and drivers allowance) was the drivers monthly salary plus cost of average quantity of diesel used by a truck per month. Meanwhile the production target of the drinks considered are the average production quantity of the chosen six most demanded drinks (chosen for the purpose of this illustration) in southern Kaduna in 2018 from the sales records. Table 2 contains the monthly average quantity of each raw material used in producing the drinks. This was obtained by dividing the number of brews of these raw materials in a month by the quantity of drinks (in truckload) produced. In the first instance, the lexicographic GP model as presented in Iwuji and Acha (2018) was used to solve the Nigerian Breweries Goal programming problem to see if the solution without the meta-goals is satisficing. The first priority of the achievement function for this model is to minimize the underachievement of the monthly profit target for all the drinks as well as producing the desired monthly targets for each of the drinks (Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export). The second priority is to minimize the overachievement of the distribution cost target. The summary of the solution obtained is presented in table 3. The solution obtained with the lexicographic GP model is unsatisfactory as only one out of the eight goals were achieved and also the sum of percentage deviation from all the goals' targets as well as the percentage deviations from some individual goals are considered very large and unrealistic. Hence the lexicographic meta-goal GP model is applied in order to

obtain a much more satisfying solution. The following three meta-goals are considered. Meta-goal 1: The sum percentage maximum deviation from all the goals should be at most 40%. Meta-goal 2: The maximum percentage of underachievement of the profit and production target of each drink goals should be at most 15%. Meta-goal 3: For the six goals on minimum production target for each of the six different drinks, at most one should be unsatisfied. The maximum bounds of 40%, 15% and at most one unsatisfied drink production target goal for meta-goal 1, 2 and 3 respectively are target margins set by management of the system in line with its desired objectives. So their values depends on management's decisions at a given point in time. We use the bounds given above for the purpose of this illustration. The meta-goals are set in the lexicographic meta-achievement function with the following priorities;

Priority 1: The maximum percentage of negative deviation from the profit goal and production target of drink goals should be at most 15%.

Priority 2: From the six goals on minimum production target for each of the six drinks, at most one should be unsatisfied.

Priority 3: The percentage maximum deviation from all the goals should be at most 40%.

These percentages deviations as well as the number of unsatisfied target for drinks are allowable deviation values set by management for each meta-goal. The system constraints of the formulated model (equation 3) which contains the raw material used in production of the drinks are presented in Table 2. This is the quantity of raw materials used in producing each drink out of total available quantity for the month. Also since the model is mixed-integer some goal's targets were strictly integer constrained while some were not. The number of truckload of drinks to be produced is strictly integer constrained as they are distributed in full while the profit and cost where not integer constrained since they can take continuous values. The solution of the Meta-GP model with lexicographic meta-achievement function and mixed integer variables is presented in the next subsection

Table 1: Estimated production capacity (per 700 crate capacity truck), profit per truckload and distribution cost for the drinks per month

	Star	Gulder	Maltina	Fayrouz	Goldsberg	33 Export	Estimated total
Production capacity (monthly)	506	350	102	420	72	264	
Profit per truckload (in Naira)	159288.77	172518.26	165519.34	125128.6	127718.94	136818.7	250,000,000
Distribution cost per truckload (in Naira)	18144.09	18144.09	18144.09	18144.09	18144.09	18144.09	31,008,250

Source: Based on calculation of estimates obtained from NBPlc Kaduna factory.

Table 2: Quantity of major raw materials used for production of drinks in a month in kg

Product	Malted sorghum	Malted barley	White sorghum	Sugar
Star	81370	124820	218040	3950
Gulder	41410	63550	111110	0
Maltina	7000	3990	26250	30100
Fayrouz	0	26436	0	96900
Goldberg	0	79500	197160	0
33 export	0	64260	154980	0
Total	129780	362556	707540	130950

Source: NBPlc Kaduna factory production department.

3.2 Results and interpretation

The summary of the solution of the Nigeria Breweries goal programming problem with a mixed-integer lexicographic GP model as used Iwuji and Acha (2018) using the LINGO optimization software is presented in table 3. The solution is considered unsatisfactory due to the large deviations from the goals' target values as the sum of percentage deviations from all the goal's target is obtained as 248.2%. This result is considered to be unrealistic for reasons which may include production capacity of the factory. For instance in the solution

we had 175.5% deviation from Maltina drink production target giving an achieved production value of 281 trucks instead of the company's target of 102 trucks. The 281 trucks monthly production might not be achieved due to production capacity of the factory. Hence the desire for a better solution.

Table 3: Summary of LINGO solution of Mixed-integer Lexicographic GP model

Priority	Goals	Minimum Target Level	Achieved value	Goal achievement	Deviation from target (%)
Priority 1	Profit goal	250,000,000	250,250,020	Achieved	0.1%
Priority 2	Star production target goal	506 trucks	506 trucks	Achieved	0%
	Gulder production target goal	350 trucks	529 trucks	Achieved	51.1%
	Maltina production target goal	102 trucks	281 trucks	Achieved	175.5%
	Fayrouz production target goal	420 trucks	420 trucks	Achieved	0%
	Goldsberg production target goal	72 trucks	72 trucks	Achieved	0%
	33 export production target goal	264 trucks	264 trucks	Achieved	0%
Priority 3	Distribution cost goal	31,008,250 Naira	24,346,000 Naira	Achieved	21.5%

The summary of the solution of the Lexicographic Meta-GP model with mixed-integer variables using LINGO optimization software is presented in Table 4.

Table 4: Summary of LINGO solution of meta-GP model

	Goals	Target Level	Achieved value	% Deviation	Goal achievement
Initial goals	Profit goal	250,000,000 Naira	249,998,544.2 Naira	0.0006% (-ve deviation)	Not achieved
	Star production target goal	506 trucks	506 trucks	0% (No deviation)	Achieved
	Gulder production target goal	350 trucks	350 trucks	0% (No deviation)	Achieved
	Maltina production target goal	102 trucks	102 trucks	0% (No deviation)	Achieved
	Fayrouz production target goal	420 trucks	420 trucks	0% (No deviation)	Achieved
	Goldsberg production target goal	72 trucks	72 trucks	0% (No deviation)	Achieved
	33 export production target goal	264 trucks	264 trucks	0% (No deviation)	Achieved
	Distribution cost goal	31,008,250 Naira	31,007,921.2 Naira	0.001% (-ve deviation)	Achieved
Meta goals	Priority 1	$\leq 15\%$	$\alpha_2 = 0, \beta_2 = 0$		Achieved
	Priority 2	≤ 1	$\alpha_3 = 0, \beta_3 = 0$		Achieved
	Priority 3	$\leq 40\%$	$\alpha_1 = 0.2026, \beta_1 = 0$		Achieved
			$\lambda = 0$		

The solution of the Lexicographic Meta-GP model is a compromised solution due to the presence of many conflicting goals and constraints. Nevertheless we can see that seven out of the eight initial goals were achieved while all the three meta-goals were achieved. For the initial goals we can see that the targets were achieved without any significant deviations thereby indicating a better result than that obtained with the lexicographic GP model. For the meta-goals, the priority 1 meta-goal which was that the maximum percentage of negative deviation from the profit and production target of drinks goals should be at most 15% was achieved as the total percentage deviation from the profit and production target of each drinks goals is 0.0006%. This means a deviation of almost zero hence the values $\alpha_2 = 0, \beta_2 = 0$. The priority 2 meta-goal which is that at most one out of the six drinks' production targets should be unsatisfied was also achieved as the target of all the six drinks were met without any deviation. Hence the values of $\alpha_3 = 0$ and $\beta_3 = 0$ as seen. The priority 3 meta-goal was also achieved as the maximum percentage deviation

from all the goals is not more than 40%. The sum of the negative deviations from the respective goals' targets is 0.002%. Hence the value of $\alpha_1 = 0.2026$ and $\beta_1 = 0$. Also the value $\lambda = 0$ indicates that the maximum deviation from the goal targets is approximately zero as observed. So we can conclude that the meta-goal programming Lexicographic achievement function model gives a more satisficing solution towards management's objectives than the case of Lexicographic GP model which had too large deviations.

3.3 Model and result validation

The solution obtained for the Meta-GP model with lexicographic achievement function is now checked to verify the presence of lexicographic redundancy. To check for lexicographic redundancy the model is solved but taking the priority levels in the meta-achievement function from the first one alone and then observe the solution. Then the second priority level is added to the first and the solution observed again. Lastly the third priority level is added to the first and second and the final solution is observed. If the solution for the case of the first priority level alone gives a more satisficing solution than when the other priority levels are added, we then conclude that the solution is lexicographic redundant. This means additional priority levels from the first one do not improve the solution. The summary of the verification is presented in Table 5.

It can be seen that the solution of the first priority level alone is not more satisficing than when the other priority levels are added but rather keeps improving instead as the priority levels are added. So we conclude that the solution obtained for the Meta-GP model with lexicographic achievement function is not redundant. Hence the solution obtained above is valid.

Table 5: Summary for verification of lexicographic redundancy.

Goals	Solution for one priority level <i>Lex. Min d = α_2</i>	Solution for one priority levels 1 & 2 combined <i>Lex Min d = α_2, β_3</i>	Solution for one priority levels 1, 2 & 3 combined <i>Lex Min d = $\alpha_2, \beta_3, \beta_1$</i>
Profit goal	249,997,974.23 Naira	250,023,200 Naira	249,998,544.2 Naira
Star production target goal	0 trucks	0 trucks	506 trucks
Gulder production target goal	0 trucks	0 trucks	350 trucks
Maltina production target goal	0 trucks	1032 trucks	102 trucks
Fayrouz production target goal	0 trucks	784 trucks	420 trucks
Goldsberg production target goal	0 trucks	0 trucks	72 trucks
33 export target goal	0 trucks	0 trucks	264 trucks
Distribution cost goal	31,005,738.04	21,338,000	31,007,921.2 Naira

3.4 Discussion of results

The comparison of the solution of the Meta-Goal programming model with lexicographic meta-achievement function presented in this paper with that of the Meta-Goal programming model with the weighted meta-achievement function presented by Acha and Iwuji (2019) is summarized in Table 6. We can see from the comparison that whereas the sum of the deviations in the Meta-GP model with weighted meta-achievement function is 34.8%, that of the Meta-GP model with lexicographic meta-achievement function is just 0.0016%. Also while six out of the eight initial goals were achievement in the Meta-GP model with weighted meta-achievement function, for the Meta-GP model with lexicographic meta-achievement function seven goals were achieved. This goes to show that the Meta-GP model with lexicographic meta-achievement function is better than that with weighted meta-achievement function for the data presented by Acha and Iwuji (2019).

Table 6: Comparison of solution of Meta-Goal Programming models with Weighted and Lexicographic meta-achievement functions.

	Goals	Target Level	% Deviation in Weighted Meta-GP model	% Deviation in Lexicographic Meta-GP model
Goals	Profit goal	250,000,000 Naira	0.0006% (-ve deviation)	0.0006% (-ve deviation)
	Star production target goal	506 trucks	0% (No deviation)	0% (No deviation)
	Gulder production target goal	305 trucks	0.9% (+ve deviation)	0% (No deviation)
	Maltina production target goal	102 trucks	2.9% (+ve deviation)	0% (No deviation)
	Fayrouz production target goal	420 trucks	0.2% (+ve deviation)	0% (No deviation)
	Goldsberg production target goal	70 trucks	30% (+ve deviation)	0% (No deviation)
	Goldsberg production target goal	70 trucks	30% (-ve deviation)	0% (No deviation)
	33 export production target goal	264 trucks	0.8% (+ve deviation)	0% (No deviation)
	Distribution cost goal	31,008,250 Naira	0.0001% (-ve deviation)	0.001% (-ve deviation)

4. Conclusion

This paper presents a Meta-GP model with lexicographic meta-achievement function that can be used by decision makers of multi-product systems in achieving satisficing solution when the initial lexicographic GP used do not give an acceptable solution. The Meta-GP approach is considered to be more flexible than the lexicographic GP model as it allowed the decision maker to establish target values not only for the initial set of goals but for another secondary set of goals of the initial goals i.e. meta-goals. This was evident in the results obtained from the illustration using data from a production factory of Nigeria Breweries PLC. From the illustration the solution obtained using the lexicographic GP model gave results which were considered unsatisfactory and undesirable as there were very large deviations from the set targets with the sum of percentage deviations from all the goal's target been 248.2%. This of course warranted the addition of meta-goals. Hence the solution obtained with the Meta-GP model with lexicographic meta-achievement function gave a much more satisficing solution as the sum of percentage deviations from all the goal's target was 0.0016% while the seven out of the eight initial goals were achieved from the introduction of the meta-goals. Also a comparison of the solution of the Meta-GP model with lexicographic meta-achievement function with that of the Meta-GP model with weighted meta-achievement function proposed by Acha and Iwuji (2019) also showed that the while Meta-GP model with lexicographic meta-achievement function is better than the Meta-GP model with weighted meta-achievement function as its sum of percentage deviations from all the goal's target was 34.8%.

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