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Finding a Manpower System's Growth Factor under Recruitment Control at Maximum Entropy

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Abstract. It is important when studying a manpower system to be able to determine the necessary condition(s) for the growth factor to achieve maximum entropy in the manpower system configuration without necessarily violating admissible recruitment conditions when the system is controlled by recruitment. The need to provide an answer to this realistic challenging issue is included in this study. This study employs a well-known transition model based on Markov chain and the Shannon entropy formula for hierarchical manpower systems to consider the growth factor and its effects on the manpower structure. The well-known condition for maximum entropy is improved upon within the conceptual underpinning of a rectangular system configuration. The bounds for the growth factor when control by recruitment is enforced at maximum entropy are derived and results are presented for a time dependent, hierarchical manpower system. The results show that there is improvement in entropy when the system expands and falls when the system contracts. More specifically, entropy increases when the change in growth factor from one point in time to the other is negative, but it decreases when the change in growth factor is positive.

Keywords: growth factor, manpower system, Markov chain, recruitment control, Shannon entropy.

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1. Introduction

The entropy index considered herein is suitable for an open hierarchical manpower system in a transient state. The population of a manpower system is usually stratified into several exclusive homogeneous groups according to var-

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ious characteristics and attributes such as length-of-service, grades, sex, qualification, physical location of members, etc. (McClean, 1991; Nilakantan and Raghavendra, 2005). By such stratification, the manpower system is transformed into a hierarchical one. When the system is considered open, the members can leave the system with a certain probability and new memberships can be allocated to a group according to a certain recruitment policy followed by the system. In transient systems, the attributes defining the homogeneous group sizes vary with time (Nilakantan, 2015). It is particularly important to examine the changes which occur in a transient system as this can allow management to quickly reverse certain policies that do not meet the target manpower structure. Apparently, the manpower system can change at any time owing to the dynamics of recruitment, internal transitions between the grades and wastage or attrition from the system. The changes that take in the system can be described using stochastic models (for example, the Markov chain model).

Economic turbulence conditions support the consideration of control of manpower systems (think of the precarious economic environment caused by the coronavirus (COVID-19) pandemic). The control problem in manpower systems largely depends on the priority set by management on whether to control promotion or recruitment or balance attainability, desirability and promotion steadiness (Komarudin et al., 2015; Komarudin et al., 2016; De Feyter et al., 2017; Ekhosuehi, 2020). When the manpower system is stratified according to length-of-service, it is more feasible to exercise recruitment control, whereas promotion control as well as its variants is realistic only when the system is grade-specific.

Before elaborating on the entropy measure for manpower systems, the notion of growth factor as used in this paper is given. It is well-known that the population size of a manpower system may be allowed to expand, contract or remain the same. In this sense, the expected population size at time point $t+1$, $N(t+1)$, is assumed to change according to its previous value (Leeson, 1984) and in such a way that

$$N(t+1) = (1+g_{t+1})N(t), \tag{1}$$

where g_{t+1} is the growth rate at the time point $t+1$. Define $a(t) = N(t)/N(t+1)$ to be the growth factor (equivalently, $a(t) = 1/(1+g_{t+1})$). Then $a(t) > 1$ denotes contraction (i.e., growth in the negative sense), $0 < a(t) < 1$ denotes expansion (i.e., growth in the positive sense) and $a(t) = 1$ represents fixed manpower size at the time point t (cf. Vassiliou, 1984). Let data for historical periods, $t = 0, 1, 2, \cdots$, up to a point in time T, be available. For reasons of statistical convenience, a stable growth factor is assumed (i.e., the growth factor is time-invariant during the period). Then equation (1) simplifies to

$$N(t+1) = N(0)/a^{t+1}, \tag{2}$$

where $N(0)$ is the population size at the base year and a is the stable growth factor. The stable growth factor was used in earlier studies (Vassiliou, 1984; Ekhosuehi et al., 2017). Using logarithmic transformation, the expression in equation (2) can be fitted to the data for $t = 0, 1, 2, \cdots, T$.

In what follows, entropy as a measure of stability in manpower systems is presented. The stability of a manpower system is essential to its wellbeing. The famous entropy measure by Shannon (1948) with some modifications has been used as a measure of stability in Markov manpower systems (McClean and Abodunde, 1978). The basic entropy formula for a $k-$grade manpower system disaggregated into homogeneous groups according to length-of-service is given as

$$H = -\frac{1}{\ln k} \sum_{j=1}^{k} q_j \ln q_j, \tag{3}$$

where q_j is the steady-state relative size of grade j. When $q_1 = 1$, $q_2 = q_3 = \cdots = q_k = 0$, $H = 0$; when $q_1 = q_2 = q_3 = \cdots = q_k = 1/k$, $H = 1$; and for intermediate values, the entropy measures the degree of experience which will be present in steady-state. When the entropy is characterised by a steady decrease in value, it would imply that the manpower system is increasingly less stable. This study refers to the entropy value at $H = 1$ when $q_1 = q_2 = q_3 = \cdots = q_k = 1/k$, as the maximum entropy condition since it is consistent with the result of maximising equation (3) subject to $\sum_{j=1}^{k} q_j = 1$.

The effect of the growth factor on the entropy of a $k-$echelon manpower system, where recruitment levels are set to compensate for wastage and to achieve the desired change in the manpower strength, is considered. This was not a major consideration in McClean and Abodunde (1978) and Vassiliou (1984). One of the basic approaches in the development of the entropy formula for manpower systems is placing a condition on the system to expand by way of an assumption (Vassiliou, 1984). This assumption plays a very important role in getting the steady state configuration of the system. Real life applications of the entropy formula indicated strongly the need for system expansion (McClean and Abodunde, 1978). The stable growth factor was utilised in Vassiliou (1984) to derive the maximum entropy condition for a manpower system in steady state.

In evaluating the stability of a graded manpower system, it is useful to consider the growth factor, which results from the recruitment levels. This is because recruitment can be carefully controlled to achieve desirable changes in the manpower strength and alleviate skewness in the manpower structure (Leeson, 1984). The perturbing issue is whether to allow the system to expand or not and how such consideration would affect its configuration. Taking the concept of control of manpower systems into account, there is a strong need to verify whether the growth factor is sufficient for an admissible recruitment strategy under the maximum entropy condition. It is worth to focus on these issues, since once instability is established it may lead to trauma (cf. McClean and Abodunde, 1978). Perhaps it is better to compute the entropy on a year-ahead basis so as to monitor the stability in the manpower system at different growth rates with a view to taking evasive action before it is too late. In this way, entropy is used as a diagnostic technique for the manpower system. The need to provide answers to these realistic challenging issues is included in the present study. Even more restriction on the growth factor is obtained under a more realistic approach, rather than making such a restriction as an assumption as in previous studies (Vassiliou, 1984). Viewed from this perspective, this study aims to determine the growth factor that is commensurate to an admissible recruitment strategy under maximum entropy.

For the readability of this paper and in order to get some more depth useful to the study of growth factor, a snapshot on the use of Markov chain to model manpower systems is provided in Sections 2. Thereafter, the condition for maximum entropy is established in Section 3. Following that an illustrative example using data for a manpower system is provided in Section 4. Section 5 concludes the paper.

2. The Markov Manpower System

When individuals in a manpower system are aggregated, the data follow probabilistic patterns which may readily be quantified for the purpose of mathematical modelling (McClean, 1991). Against this background, the Markov chain model with fixed transition probabilities, p_{ij}, is constructed for graded manpower systems. From the point of view of practice, the probabilities, p_{ij}, are estimated from historical manpower data, which are available to management. There is an extensive body of literature regarding the state-of-the-art of Markov modelling and control of manpower systems (Guerry, 2008; Guerry and De Feyter, 2012; Dimitriou et al., 2013; Udom, 2014; Komarudin et al., 2015; Nilakantan, 2015; Dimitriou et al., 2015). A comprehensive treatment of earlier works on how changes take place in a manpower system including stationarity and control is given by Bartholomew et al. (1991).

An interesting idea was proposed by Georgiou and Tsantas (2002), where a training/standby class was introduced to the states of a manpower system. This idea was employed by Dimitriou and Tsantas (2009, 2010) to exercise recruitment control on a time dependent, hierarchical manpower system, wherein training classes and an auxiliary external system to cater for demand for specific skilled individuals for hiring were incorporated.

Nilakantan and Raghavendra (2005) introduced the concept of proportionality policy to manpower systems. This concept follows from the restriction on recruitment to every level of the hierarchy (except the bottom most level or base level) to be in strict proportion to promotions into that level. The proportionality restrictions dictate that the number of recruits in any transition interval should be within a fixed proportion of the number of entrants promoted to the grade from within the organisation in the same interval.

In a manpower system where members each belong to one of the mutually exclusive and exhaustive grades in $S = \{1, 2, \cdots, k\}$, the flow of individuals can be subdivided into the recruitment stream, the transitions between the grades and wastage from the system. In the grade-specific set S, the base grade is grade 1 and the uppermost grade is grade k. Considering a discrete time scale, $t = 0, 1, 2, \cdots$, the individual transitions between the grades are assumed to evolve according to a time-homogeneous Markov chain, $\mathbf{P} = (p_{ij})$, $i, j \in S$, where p_{ij} is the probability of being in grade j given that an individual was in grade i at the previous time point. An individual in grade i leaves the system in $(t, t+1)$ with probability

$$w_i = 1 - \sum_{j=1}^{k} p_{ij}, \tag{4}$$

for each $i \in S$. Assuming homogeneity in time, recruited persons are allowed to grade $i \in S$ according to a recruitment distribution $\{r_i\}$ with $r_i \geq 0$ and $\sum_{i=1}^{k} r_i = 1$. These restrictions define the admissible conditions for recruitment.

Denote the expected number of persons in grade i at time point t by $n_i(t)$ and let $N(t) = \sum_{i=1}^{k} n_i(t)$ be the total size of the population at that time. The initial grade sizes $n_i(0)$, $i \in S$, are assumed to be known. Let $q_i(t) = n_i(t)/N(t)$ be the relative size of grade i at time point t. The distribution $\{q_i(t)\}$, being emblematic of the so-called 'configuration of the system', is the manpower (or personnel) structure at the time point t. Further, let $R(t)$ be the expected number of new recruits at the time point t. For a system with high demand (as is the case in most developing countries), it is reasonable to assume that recruitment is done to meet immediate demand in such fashion that it compensates for wastage (such as retirement, death, resignation, etc.) and achieves the desired change in the manpower strength (Leeson, 1984; Vassiliou, 2015). Under this assumption, the Markov chain governing the system becomes an embedded Markov chain. The description of manpower systems based on embedded Markov chain has flourished the manpower literature (for instance, Vassiliou and Symeonaki, 1999; Georgiou and Tsantas, 2002; Ekhosuehi, 2020).

The development in the manpower system at successive points in time within the context of this study is expressed in a condensed form through the following difference equation that governs the evolution of the system's configuration:

$$q_j(t+1) = \frac{N(t)}{N(t+1)} \sum_{i=1}^{k} p_{ij} q_i(t) + \left(\frac{N(t)}{N(t+1)} \sum_{i=1}^{k} w_i q_i(t) + \frac{\Delta N(t+1)}{N(t+1)} \right) r_j, \tag{5}$$

$j \in S$, $t = 0, 1, 2, \cdots$. The difference equation is evaluated recursively at each point in time t to obtain the relative grade size, $q_j(t+1)$, at the next time point. Define $a(t) = N(t)/N(t+1)$ as earlier. Then

$$q_j(t+1) = a(t)\sum_{i=1}^{k} p_{ij}q_i(t) + \left(a(t)\sum_{i=1}^{k} w_i q_i(t) + \frac{\Delta N(t+1)}{N(t+1)}\right) r_j. \qquad (6)$$

This simplifies to

$$q_j(t+1) = a(t)\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) + (1 - a(t))\, r_j, \qquad (7)$$

$j \in S$, $t = 0, 1, 2, \cdots$. Thus, the homogeneous Markov chain for this system at hand is $\mathbf{Q} = (q_{ij})$, where $q_{ij} = p_{ij} + w_i r_j$.

3. The Bounds for $a(t)$ at Maximum Entropy

Despite the motivation for the use of entropy in thermodynamics, statistical mechanics, information theory and experimental design (Tyler, 1984; Ekhosuehi et al., 2018; Metzig and Colijn, 2020), its extension to mathematical human resource planning offers a more general, and perhaps more attractive, framework to measure stability in manpower systems. Very recently, in an interesting work by Ekhosuehi (2020), the principle of maximum entropy was used to derive transition probabilities for a stable Gani-type person-flow manpower system. The work of Ekhosuehi (2020) provides new grounds to represent entropy for a manpower system in transient state.

Early academic contribution to entropy of manpower systems studied the $k-$length-of-service-specific manpower system (McClean and Abodunde, 1978; Vassiliou, 1984). When Markov chain theory is applied to this kind of system, the transitions from one length-of-service class to the next is represented by a sub-stochastic transition matrix of the form

$$\mathbf{P} = \begin{bmatrix} 0 & p_{12} & 0 & \cdots & 0 \\ 0 & 0 & p_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & p_{k-1,k} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

In this case, it is possible for the relative size of a grade to be zero. McClean and Abodunde (1978) recognised this possibility and defined the natural logarithm of zero to be zero so that the formula (3) is defined when $q_j = 0$. In the current study, a single-characteristic manpower system is considered with the population defined by grades. The transition probabilities, p_{ij}, are replaced by $(p_{ij} + w_i r_j)$ in order to model the system that allows a loss from grade i to be associated with a gain to grade j (cf. equation (7)).

It is rather useful at this point, to write something about the entropy of a graded manpower system in a transient state. Viewed from the perspective of McClean and Abodunde (1978) and the possibility to represent entropy for a manpower system in a transient state at discrete time point $t = 0, 1, 2, \cdots$, entropy is defined for the system in equation (7) as

$$H(t+1) = -\frac{1}{\ln k} \sum_{j=1}^{k} q_j(t+1) \ln q_j(t+1). \tag{8}$$

The entropy formula attains a maximum value of one when the manpower structure is rectangular. It takes the value zero for an extreme case of pyramid-like or top-heavy structure when only either $q_1(t+1) = 1$ or $q_k(t+1) = 1$, respectively, with $\sum_{j=1}^{k} q_j(t+1) = 1$, $q_j(t+1) \geq 0$ for all $j \in S$. For intermediate values, the entropy measures the degree of experience in the manpower structure at the time point $t+1$. Making appropriate substitution of equation (7) into (8),

$$H(t+1) = -\frac{1}{\ln k} \sum_{j=1}^{k} \left(a(t) \sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) + (1 - a(t))\, r_j \right) \times$$

$$\ln \left(a(t) \sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) + (1 - a(t))\, r_j \right). \tag{9}$$

Taking the first-order derivative of $H(t+1)$ with respect to $a(t)$ and then setting the result to zero, it is verifiable that

$$\sum_{j=1}^{k} \left(\sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) - r_j \right) \ln \left(a(t) \sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) + (1 - a(t))\, r_j \right)$$

$$= 0 \tag{10}$$

with $w_i = 1 - \sum_{j=1}^{k} p_{ij}$. For sufficiency, it can be shown that the result from equation (10) yields a maximum value as the second-order derivative

$$\frac{d^2 H(t+1)}{d a^2(t)} = -\frac{1}{\ln k} \sum_{j=1}^{k} \frac{\left(\sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) - r_j \right)^2}{\left(a(t) \sum_{i=1}^{k} (p_{ij} + w_i r_j) q_i(t) + (1 - a(t))\, r_j \right)} < 0.$$

This second-order derivative reasonably calls for $0 < a(t)\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) + (1 - a(t))\, r_j < 1$, for each $j \in S$. Suppose that $a(t)\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) + (1 - a(t))\, r_j = 1$ for $j = 1$. Then, within the context of entropy, $a(t)\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) + (1 - a(t))\, r_j = 0$ for $j > 1$, which implies the absence of experience in the system.

Unmistakeably, maximum entropy is consistent with expansion of the system. To see this, take a look at $a(t)\sum_{i=1}^{k}(p_{ij}+w_i r_j)q_i(t)+(1 - a(t))\, r_j = 0$ for $j > 1$. As $(a(t) - 1)\, r_j/a(t) = \sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) \geq 0$, it is clear that $a(t) \geq 1$. This is the reason why maximum entropy may not be attained when a manpower system does not expand. We conclude that the manpower system expands for maximum entropy to be attained. Thus,

$$a(t) < 1. \tag{11}$$

In accordance with the maximum entropy condition which is known in the sequel, the entropy (9) attains its maximum value when

$$a(t)\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) + (1 - a(t))\, r_j = \frac{1}{k} \tag{12}$$

for each $j \in S$ and

$$\sum_{j=1}^{k}\left(\sum_{i=1}^{k}(p_{ij} + w_i r_j)q_i(t) - r_j\right) = 0. \tag{13}$$

Assume the case, where the management decides to adopt recruitment control and pre-set the internal transition parameters, p_{ij}, as a coping strategy. This is encountered in practice when restrictive covenants between management and labour unions are reached in the form of protectionist manpower policy, and the p_{ij}'s are pre-set to safeguard the career prospects of the already existing members of staff. It is rather unusual for $r_j < 0$ as this could introduce redundancy even when the p_{ij}'s are pre-fixed (Bartholomew et al., 1991). Taking this into consideration, equation (12) becomes an attainability problem that needs to be solved subject to the admissibility conditions, $r_i \geq 0$ and $\sum_{i=1}^{k} r_i = 1$, specified for recruitment earlier on. Suppose the recruitment control is to be executed at time $t = t^*$. Then, from equation (12), it can be verified that

$$a(t^*) \geq \frac{1}{k\sum_{i=1}^{k} p_{ij}q_i(t^*)}, \tag{14}$$

for each $j \in S$. Let

$$\mu_j(t^*) = \frac{1}{k\sum_{i=1}^{k} p_{ij} q_i(t^*)}.$$

Then the growth factor, $a(t^*)$, can be chosen as follows: if

$$\min_{j \in S} \left(\mu_j(t^*)\right) < 1$$

set

$$a(t^*) = \min_{j \in S} \left(\mu_j(t^*)\right); \tag{15}$$

otherwise, select $a(t^*) = a^* < 1$.

This proposed criterion for determining $a(t^*)$ establishes the necessary growth condition that would achieve recruitment strategy that is admissible under maximum entropy condition. Needless to say that this choice of $a(t^*)$ has taken the condition for maximum entropy an important step forward in terms of the bounds for the growth factor within the rectangular system configuration. Note that in the non-homogeneous Markov chain case, the analysis is straightforward and in the same manner it is done for the homogeneous case, except that the fundamental internal transition parameter, p_{ij}, in the relation (14) is now replaced by $p_{ij}(t^*)$.

4. Illustration

The preceding theoretical results are illustrated using the manpower system in Vassiliou (2015). This illustrative example gives an overview of the method discussed so far to the reader. Consider a manpower system with 8 grades described by a periodic embedded nonhomogeneous Markov chain, $\{\mathbf{Q}(t)\}_{t=0}^{\infty}$, where

$$\mathbf{Q}(t) = \begin{bmatrix} \mathbf{Q}_1(t) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2(t) & \mathbf{0} \\ \mathbf{T}_1(t) & \mathbf{T}_2(t) & \mathbf{T}(t) \end{bmatrix}$$

with

$$\mathbf{Q}_1(t) = \begin{bmatrix} 0 & 0.9 - \frac{1}{3t^4+1} & 0 & 0.1 + \frac{1}{3t^4+1} \\ 0.8 - \frac{1}{5t} & 0 & 0.2 + \frac{1}{5t} & 0 \\ 0 & 0.7 - \frac{3}{7t} & 0 & 0.3 + \frac{3}{7t} \\ 0.05 + \frac{1}{8t^2} & 0 & 0.95 - \frac{1}{8t^2} & 0 \end{bmatrix}, \quad \mathbf{Q}_2(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{T}_1(t) = \begin{bmatrix} 0 & 0.1 + \frac{1}{3t^2} & 0 & 0.1 + \frac{1}{6t+1} \\ 0 & 0 & 0 & 0.2 + \frac{2}{11t^2} \end{bmatrix}, \quad \mathbf{T}_2(t) = \begin{bmatrix} 0.1 & 0.4 - \frac{1}{3t^2} \\ 0.2 - \frac{3}{15t^3} & 0.2 + \frac{1}{7t^3} \end{bmatrix},$$

$$\mathbf{T}(t) = \begin{bmatrix} 0.3 - \frac{1}{6t+1} & 0 \\ 0.3 - \frac{2}{11t^2} & 0.1 + \frac{1}{15t^3} \end{bmatrix}.$$

The recruitment probabilities are given in the following vector

$$\mathbf{P}_0(t) = \begin{bmatrix} 0 & 0.2 - \frac{1}{35t} & 0 & 0.2 + \frac{1}{35t} & 0 & 0.2 - \frac{1}{7t} & 0 & 0.4 + \frac{1}{7t} \end{bmatrix}.$$

The initial population structure of the system is given by the vector

$$\mathbf{n}(0) = \begin{bmatrix} 50 & 80 & 120 & 150 & 200 & 300 & 50 & 150 \end{bmatrix}$$

and the initial number of memberships in the system is $N(0) = 1100$. The number of memberships afterwards are: $N(1) = 1150$, $N(2) = 1190$, $N(3) = 1220$, $N(4) = 1250$, $N(5) = 1310$ and $N(6) = 1400$. According to Vassiliou (2015) the dynamics described by $\{\mathbf{Q}(t)\}_{t=0}^{\infty}$ is analogous to the manpower system of an investment bank with eight grades where the first four grades represent sections of traders, the next two grades denotes back office staff and the remaining two grades being training grades.

At first glance, the number of memberships over the period shows that the system expands over time. Upon more accurate inspection by fitting the trend in the number of memberships against time, the stable growth factor is found to be $a = \exp(-0.0369) = 0.9638$, and is adjudged (by the p-value) to be significant at 1% level. Assume that the management decides to pre-set the internal transition probabilities for $t = 7$, so that $t^* = t - 1$, as follows: $p_{12}(6) = 0.6$, $p_{14}(6) = 0.1$, $p_{21}(6) = 0.8$, $p_{23}(6) = 0.1$, $p_{32}(6) = 0.5$, $p_{34}(6) = 0.2$, $p_{41}(6) = 0.05$, $p_{43}(6) = 0.9$, $p_{56}(6) = 1$, $p_{65}(6) = 1$, $p_{74}(6) = 0.1$, $p_{75}(6) = 0.1$, $p_{76}(6) = 0.2$, $p_{77}(6) = 0.3$, $p_{84}(6) = 0.1$, $p_{85}(6) = 0.2$, $p_{86}(6) = 0.1$, $p_{87}(6) = 0.2$, $p_{88}(6) = 0.1$.

We proceed further to monitor the entropy of the system on a year-to-year basis and verify whether the expansion in the system could guarantee an admissible recruitment strategy with the given internal transition probabilities at $t = 7$. To use entropy as a means of monitoring changes in the system, the growth factor is obtained for each $t = 0, 1, 2, 3, 4, 5, 6$ using $a(t) = N(t)/N(t+1)$ and the formula (9) is employed to compute the entropy values. Table 1 shows the variation in entropy alongside with the growth factor for each time point. The results in Table 1 show that a small variation in size affects entropy. There is improvement in entropy when the system expands, but entropy falls when the system contracts. More specifically, entropy increases when the change in growth factor from one point in time to the other is negative, but entropy decreases when the change in growth factor is positive. Thus, a marginal increase in the growth factor, $\Delta a(t) \uparrow$, is characterised by a marginal decrease in entropy, $\Delta H(t) \downarrow$. Conversely, a marginal decrease, $\Delta a(t) \downarrow$, would result to a marginal increase in entropy, $\Delta H(t) \uparrow$. The manpower system is increasingly less experienced for $t = 1, 2, 3, 4$. At $t = 5, 6$, there is improvement in the level of experienced manpower present in the system.

Table 1: Fluctuations in entropy over time

t	$a(t-1)$	$\Delta a(t) \uparrow\downarrow$	$H(t)$	$\Delta H(t) \uparrow\downarrow$
1	0.9565	–	0.8948	–
2	0.9664	$\Delta a(t) \uparrow$	0.8707	$\Delta H(t) \downarrow$
3	0.9754	$\Delta a(t) \uparrow$	0.8607	$\Delta H(t) \downarrow$
4	0.9760	$\Delta a(t) \uparrow$	0.8497	$\Delta H(t) \downarrow$
5	0.9542	$\Delta a(t) \downarrow$	0.8553	$\Delta H(t) \uparrow$
6	0.9357	$\Delta a(t) \downarrow$	0.8687	$\Delta H(t) \uparrow$

The arrow $\uparrow$ denotes a positive change and $\downarrow$ represents a negative change.

We determine $a(6)$ as follows. Using the pre-set probabilities, the values of $\mu_j(t^*) = \mu_j(6)$ for each $j \in S$ are obtained as: $\mu_1(6) = 0.8866$, $\mu_2(6) = 0.9387$, $\mu_3(6) = 1.6908$, $\mu_4(6) = 3.3289$, $\mu_5(6) = 0.4543$, $\mu_6(6) = 0.5499$, $\mu_7(6) = 15.7431$, $\mu_8(6) = 42.8082$. What is noticeable is that none of the pre-existing growth factors in Table 1 could guarantee admissible recruitment strategy since they all exceed the minimum, $\min_{j\in S}(\mu_j(t^*)) = \mu_5(6) = 0.4543$. As a result, the system should engage more new members (recruits) by selecting $a(6)$ to be 0.4543.

5. Conclusion

The application of Shannon entropy to manpower systems has yielded satisfactory results in describing the system in steady state. Nonetheless, certain instances may arise where the entropy may have to be computed on a year-to-year basis. This study employed the notion of Shannon entropy to select the growth factor consistent with recruitment control at maximum entropy within the theoretical framework of Markov chains. In this regard, the condition for maximum entropy was taken as an important step forward in terms of the bounds for the growth factor within the rectangular system configuration. The readers have been familiarised with the possibility of using the growth factor as a proactive action strategy under recruitment control. By viewing the embedded Markov chain system in the literature, the entropy pattern for the system was successfully monitored vis-á-vis the most appropriate growth factor under recruitment control. The results indicate that a continuous decrease in the growth factor would lead to a steady increase in entropy and vice versa. Thus the paper is a significant addition to the existing methodologies in the literature on manpower systems. In practice, however, one can have good reasons to prefer a top-heavy or pyramidal structure. Further investigation of the entropy of these kind structures is worthwhile as the present study did not consider the entropy of such manpower systems.

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