

Spatial Econometric Analysis with Application of Phillips Curve on Nigerian Economy

N. A. Okoro-Ugochukwu¹, and M. O. Adenomon^{*, 1, 2, 3}

¹*Department of Statistics, Nasarawa State University, Keffi, Nigeria & NSUK-LISA Stat Lab, Nasarawa State University, Keffi, Nigeria*

²*Chair, International Association of Statistical Computing (IASC) African Members Group*

³*Foundation of Laboratory for Econometric and Applied Statistics of Nigeria (FOUND-LEAS-IN-NIGERIA)*

(Received: 14 June 2021; Accepted: 26 May 2022)

Abstract. In recent times, spatial econometrics application is given reasonable attention as it deals with data of spatial type either in cross-sectional or panel form. This study examined application of Phillips Curve on the Nigerian Economy using 2018 cross-sectional data of consumer price index (CPI) as inflation rate and unemployment rate data sourced from the National Bureau of Statistics (NBS), Nigeria. Results from the Ordinary Least Squares (OLS) confirmed a negative relationship between inflation and unemployment in Nigeria though not significant while autocorrelation is present in the estimated model at 10% level of significance. The Moran I statistic for spatial autocorrelation test is only significant at 10% while the Monte-Carlo simulation of Moran I statistic at 10,000 simulations revealed the presence of spatial autocorrelation at 1% level of significant. Spatial Lag Model (SLM), Spatial Error Model (SEM) and Spatial Autoregressive with autoregressive error structure (SARAR) were applied in this study. The result from the spatial lag model shows a unit increase in unemployment leads to a decrease of 0.0011 in inflation rate. Lastly, a unit increase of unemployment in one state of Nigeria produces a total impact of reduction of 0.0014 in inflation rate. The findings support Phillips Curve but the relationship is not significant in the case of the Nigerian Economy.

Keywords: Phillips Curve, Cross-sectional, Inflation, Unemployment, OLS, SLM, SEM, SARAR, Models

Published by: Department of Statistics, University of Benin, Nigeria

*Corresponding author. Email: adenomonmo@nsuk.edu.ng

1. Introduction

The occurrences of the effect of spatial autocorrelation in regression model gives rise to the development of spatial econometrics in order to obtain reliable model and estimates. Spatial econometrics is a subfield of econometrics that deals with spatial interaction (spatial autocorrelation) and spatial structure (spatial heterogeneity) in regression models for cross-sectional and panel data (Baltagi, 2001; Arbia, 2014; Okoro-Ugochukwu and Adenomon, 2021). Another concept that make spatial econometrics attractive is the First law of Geography. The first law of geography states that “everything is usually related to all else but those which are near to each other are more related when compared to those that are farther away as stated by Waldo Tobler in 1970 (Dempsey, 2014). In addition, Tobler’s first law of geography is one of the key reasons while “spatial is special” (Li *et al.*, 2014). This first law bring to play the concept of spatial autocorrelation and spatial econometrics.

The study examined the application of Philips curve on the Nigerian Economy using some spatial models such as Spatial Regression models (Spatial Lag Model (SLM); Spatial Error Model (SEM) and Spatial Auto-regressive with additional Auto-regressive error structure (SARAR). Phillips curve rule states that, as unemployment level increases, the rate of inflation falls. Thus, there exist a trade-off between inflation and unemployment; the higher the inflation rate, the lower the unemployment rate. Previous empirical studies are as follows: Samuelson and Solow (1960) were the first researchers who supported the Phillips hypothesis in their paper for US supporting the negative relationship between unemployment and inflation Later, Phelps (1967) and Friedman (1968) criticized the Phillips hypothesis and mentioned that there is no trade-off relationship between unemployment and inflation. Meanwhile, Lucas (1976) strongly opposed the proposition of the existence of the Phillips curve, supporting that there could be a trade-off relationship between unemployment and inflation, provide that policy makers have not created a situation where high inflation is paired with low unemployment.

Some studies such as Furuoka (2007) explored the connection between unemployment rate and inflation rate in Malaysia. The data from 1975 to 2004 were analyzed using the Johansen co-integration and error correction model tests. The results showed that there is a connection running from the unemployment rate to inflation rate in both short and long run. Therefore, these findings confirmed the existence of the Phillips curve in Malaysia. Katria *et al.* (2011) selected eight SAARC countries including Afghanistan, Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan and Sri Lanka, and six expected countries of SAARC, namely, Republic of China, Russia, Indonesia, Iran, Myanmar and South Africa to analyze the connection between inflation and unemployment for the same period of 1980-2010. The analysis was based on the multiple regression and the results indicated that there is a negative connection between inflation and unemployment rate in the SAARC Countries. Haug and King (2014) estimated the long run vertical Phillips curve in the US using a band-pass filter approach. They found strong evidence that a positive relationship exists in which inflation leads to unemployment in 3–3.5 years, occurring in cycles that last from 8 to 25 or 50 years.

Karahan *et al.* (2013) examined the connection between unemployment and in-

flation in Turkey for the monthly data from 2006 to 2011. The Autoregressive Distributed Lag (ARDL) Bounds test was conducted and the findings showed that unemployment negatively affects inflation in the short run but there is no causal connection in the long run. Umoru and Anyiwe (2013) in their study of the dynamics of inflation and unemployment in Nigeria using a Vector Error Correlation Model over a period of 27 years discovered the existence of stagflation in Nigeria over the studied period. They discovered that Nigerian economy is battling with a shocking rate of inflation alongside a severe recession as the unemployment rate had risen astronomically. They conclude that the Nigerian economy is at the cross-road. Kogid *et al.* (2011) while studying inflation-unemployment trade-off relationship in Malaysia using three robust econometrics methods of ARDL bonds testing technique, ECM based ARDL and Toda-Yamamoto (1995) techniques for the period 1975 – 2007. Their empirical result demonstrates that a long run co-integration relationship exist between inflation and unemployment and a unidirectional causal relationship running from inflation to unemployment. They conclude that there was an evidence of inflation-unemployment trade-off relationship in Malaysia.

Dumlao (2012) investigated the relationship between price and unemployment in terms of Philips Curve for Malaysia, the Philippines, Singapore and Thailand which shows a weak fit. The work further examined the relationship from the supply curve (using dynamic price and dynamic unemployment) which then shows a better fit of negative relationship between dynamic price and dynamic unemployment. Odo *et al.* (2017) conducted a similar study titled understanding the relationship between unemployment and inflation in Nigeria between 1980 – 2015. They modeled unemployment as a function of inflation and adopted causality test, VECM and Johansen co-integration tests in their analysis. Their findings indicate that inflation had a significant impact on unemployment in Nigeria both in the short and long-run. They maintain that increase in government expenditure reduce unemployment and such government spending creates employment to the extent that inflation remains within the single digit ambit. Iyeli and Ekpung (2017) in a similar investigation on price expectations and the Philips curve hypothesis in the Nigerian economy made use of Parsimonous Error Correction Model and Johansen method of co-integration. Their result revealed the prevalence of a direct (positive) relationship between inflation and unemployment in Nigeria and thus invalidates the Philips curve hypothesis of an inverse (negative) relationship.

Saad and Salim (2017) undertook a study on inflation and unemployment in Nigeria using an ARDL model approach. They made use of annual time series data of 1977 to 2011 in their analysis. The result of the co-integration result indicates that a long-run relationship exists between the variables of inflation and unemployment in Nigeria. Their finding supports the applicability of Philips curve hypothesis in Nigeria and as such contradicts the popular idea of the co-existence of unemployment and inflation in the country. Edeme (2018) in his study provided an empirical insight into Nigeria's Non-Accelerating Inflation Rate of Unemployment (NAIRU) made use of annual time series data between 1972 and 2015 obtained from the statistical bulletin of CBN. The Ordinary Least Squares (OLS) method was adopted to ascertain if Philips curve postulate exists in Nigeria both in the short-run and long-run. The emanating evidence

from the empirical analysis established the existence of a negative but insignificant relationship between inflation and unemployment both in the short-run and long-run in Nigeria. Orji *et al.* (2015) examined the inflation and unemployment nexus in Nigeria using ARDL model on annual data covering 1981 to 2011. Their results indicate that unemployment and inflation rate in Nigeria has a significant positive relationship. Chuku *et al.* (2017) examined the problem of testing for the stability and persistence of the Phillips curve for Nigeria when there are non-stationarities in the marginal distribution of the regressors. Using quarterly data from 1960 to 2019, the study found out that, unlike Andrews asymptotic p-values, inference based on Hansen's hetero-corrected bootstrap technique supports the hypothesis of a structural break in the inflation dynamics in Nigeria. Uche (2019) examined the relationship between unemployment and inflation rate in Nigeria using Fully Modified Ordinary Least Squares (FMOLS) regression on annual data covering 1981 to 2017 collected from Central Bank of Nigeria (CBN) and National Bureau of Statistics (NBS). The results revealed inverse relationship between unemployment and inflation rate in Nigeria.

Abu (2019) examined the Phillips curve hypothesis and its stability in Nigeria from 1980 to 2016 using ARDL, Fully Modified Ordinary Least Squares (FMOLS), Dynamic Ordinary Least Squares (DOLS), Static ordinary Least squares (OLS) and Canonical co-integration regression (CCR). The results from ARDL, FMOLS, DOLS, Static OLS and CCR indicated that there is a trade-off relationship between the variables and higher unemployment leads to lower inflation in the long run. Efayena and Olele (2020) validated Phillips curve hypothesis in Nigeria using Generalized Method of Moments (GMM) and Canonical cointegrating regression (CCR) methods on quarterly data of inflation and unemployment between 2010 and 2018. The study validate the presence of a Phillips in the Nigerian economy. Daniel *et al.* (2021) explored the relationship between inflation and unemployment using Vector Autoregressive and Error Correction Models on secondary data from CBN and World Bank covering 1981 to 2020. The results indicates no significant relationship between inflation and unemployment in Nigeria. Ayinde *et al.* (2021) investigated the New Keynesian Phillips curve (NKPC) extended by Hybrid NKPC (H-NKPC) in Nigeria using quarterly data from 2000Q1 to 2018Q4. Results from Generalized Methods of Moments (GMM) supports the H-NKPC which implied that lagged inflation is a significant part of inflation determinants in Nigeria.

From the forgoing, most studies have not pay much attention of the study application of Philips curve on developing economy such as Nigerian Economy using spatial regression model. Hence the need for this present study. This study examined the application of Philips curve on the Nigerian Economy using Spatial Regression models (Spatial Lag Model (SLM); Spatial Error Model (SEM) and Spatial Auto-regressive with additional Auto-regressive error structure (SARAR).

2. Materials and Method

In this study, we used a cross sectional data namely unemployment and inflation rates with spatial feature from the National Bureau of Statistics Bulletin (NBS) 2018. The study used inflation rate as dependent variable and unemployment

rate as independent variable (see Table 1 at the appendix).

2.1 Model Specification

Spatial models such as Spatial Lag Model, Spatial Error Model and Spatial Autoregressive with additional Auto-regressive error structure were employed in this study.

The following models such as Spatial Lag Model, Spatial Error Model and Spatial Autoregressive with additional Auto-regressive error structure are employed to account for spatial auto-correlation error that is inherent in cross-sectional data. We would discuss different specifications of linear Spatial Econometric models which can be considered once the hypothesis of no Spatial autocorrelation in the disturbances is violated.

The general condition for the applicability of Ordinary Least Squares is given by the equation below:

$$y = \lambda W y + X \beta_{(1)} + W X \beta_{(2)} + u \quad |\lambda| < 1 \quad (1)$$

$$u = \rho W u + \varepsilon \quad |\rho| < 1 \quad (2)$$

with the X known to be a matrix of non-stochastic regressors, W is the weighted matrix exogenously given, $e|X \approx i.i.d. N(0, \sigma_{\varepsilon n}^2 I_n)$, $u | X \approx i.i.d. N(0, \sigma_{\varepsilon n}^2 I_n)$ and $\beta_{(1)}$, $\beta_{(2)}$, λ and ρ are the parameters to be estimated. The restrictions on the parameters, λ and ρ hold if W is row-standardized.

(1) considers the spatially lagged variable of the dependent variable y as one of the regressors which may also contain spatially lagged variables of some or all of the exogenous variables (the term WX). While (2) considers a spatial model for the stochastic disturbances. (1) can also be written as:

$$y = \lambda W y + Z \beta + u \quad |\lambda| < 1. \quad (3)$$

Haven defined the matrix of all regressors, current and spatially lagged, as $Z = [X, WX]$ and the vector of regression parameters as $\beta = [\beta_{(1)}, \beta_{(2)}]$

This model was termed Spatial Auto Regressive with additional Auto Regressive error structure as SARAR (1,1) by Kelejian and Prucha (1998) and encompasses several spatial econometric models. In particular we considered three remarkable cases:

- (i) $\lambda = 0$, $\rho \neq 0$ known as Spatial Lag Model (SLM)
- (ii) $\lambda \neq 0$, $\rho = 0$ known as Spatial Error Model (SEM)
- (iii) $\lambda \neq 0$, $\rho \neq 0$ the complete model (SARAR)

We will review these three cases in the following sections. Before doing this, let us consider a general condition on the model's parameters.

With (1) and (2) written as:

$$(I - \lambda W)y = X\beta_{(1)} + WX\beta_{(2)} + u$$

$$y = (I - \lambda W)^{-1}[X\beta_{(1)} + WX\beta_{(2)} + u] \quad (4)$$

and

$$u = (I - \rho W)^{-1}\varepsilon \quad (5)$$

provided that the two inverse matrices exist. Using the Gerschgorin (1931) theorem Kelejian and Prucha (1998) proved that, when the W matrix is row-standardized, both inverse matrices exist if $|\lambda| < 1$ and $|\rho| < 1$.

2.1.1 The Spatial Lag Model (SLM)

When $\lambda \neq 0$ and $\rho = 0$ the model becomes

$$y = \lambda Wy + Z\beta + u \quad |\lambda| < 1,$$

With $u | X \approx i.i.d. N(0, \sigma_{\varepsilon}^2 I_n)$. This model is referred to in the literature as the Spatial Lag Model (SLM) (Anselin, 1988; Arbia, 2006). In this case, a problem of endogeneity emerges in that the spatially lagged value of y is correlated with the stochastic disturbance.

Given that

$$(I - \lambda W)y = Z\beta + u \text{ and } y = (I - \lambda W)^{-1}Z\beta + (I - \lambda W)^{-1}u$$

so that the correlation between the lagged term Wy and the error can be expressed as

$$\begin{aligned} E[(Wy) u^T] &= E[W(I - \lambda W)^{-1}Z\beta + (I - \lambda W)^{-1}u]u^T \\ &= W(I - \lambda W)^{-1}Z\beta E(u^T) + W(I - \lambda W)^{-1}E(uu^T) \\ &= \sigma_{\varepsilon}^2 W(I - \lambda W)^{-1}, \quad I \neq 0 \end{aligned} \quad (6)$$

so, in the presence of endogeneity, a GLS procedure cannot be employed. This study employed Maximum Likelihood (Arbia, 2014).

Maximum Likelihood (ML) Estimator

Let us consider again the full model contained in (1) and (2), we thus have

$$y = \lambda Wy + X\beta_{(1)} + WX\beta_{(2)} + u \quad |\lambda| < 1$$

$$u = \rho W u + \varepsilon \quad |\rho| < 1$$

$$\mathcal{E} \mid X \approx i.i.d. N(0, \sigma_\varepsilon^2 I_n).$$

From (1), we have

$$E(y) = (I - \lambda W)^{-1} Z \beta$$

and also,

$$E(yy^T) = E[(I - \lambda W)^{-1} (I - \rho W)^{-1} \mathcal{E} \mathcal{E}^T (I - \lambda W)^{-T} (I - \rho W)^{-T}]$$

$$= \sigma_\varepsilon^2 (I - \lambda W)^{-1} (I - \rho W)^{-1} (I - \lambda W)^{-T} (I - \rho W)^{-T}$$

$$= \sigma_\varepsilon^2 \Omega$$

hence, maintaining the hypothesis of normality on the disturbances, we have:

$$y \approx N[(I - \lambda W)^{-1} X \beta; \sigma_\varepsilon^2 \Omega]$$

the likelihood can be derive as follows

$$L(\sigma^2, \rho, \lambda, \beta; y) = \text{const} (\sigma_\varepsilon^2)^{-\frac{n}{2}} |I - \lambda W| |I - \rho W| \\ \times \exp \left\{ -\frac{1}{2\sigma_\varepsilon^2} [y - (I - \rho W)^{-1} Z \beta]^T \times \Omega^{-1} [y - (I - \rho W)^{-1} Z \beta] \right\}$$

and the log-likelihood can be expressed as

$$l(\sigma^2, \rho, \lambda, \beta; y) = \text{const} - \frac{n}{2} \ln(\sigma_\varepsilon^2) + \frac{1}{2} \ln |I - \lambda W| + \ln |I - \rho W|$$

$$- \frac{1}{2\sigma_\varepsilon^2} [y - (I - \lambda W)^{-1} Z \beta]^T (I - \lambda W)^T$$

$$\times [I - \rho W]^T (I - \rho W) (I - \lambda W) \times [y - (I - \rho W)^{-1} Z \beta]$$

And since $(I - \lambda W) [y - (I - \rho W)^{-1} Z \beta] = (I - \lambda W)y - Z \beta$, we eventually obtain:

$$l(\sigma^2, \rho, \lambda, \beta; y) = \text{const} - \frac{n}{2} \ln(\sigma_\varepsilon^2) + \frac{1}{2} \ln |I - \lambda W| + \ln |I - \rho W|$$

$$-\frac{1}{2\sigma_{\varepsilon}^2}[(I - \rho W)(y - Z\beta - \lambda Wy)]^T \times (I - \rho W)(y - Z\beta - \lambda Wy)]$$

2.1.2 The Spatial Error Model (SEM)

In this study, the Spatial Error Model (SEM) was estimated using Maximum Likelihood (ML) and Feasible Generalized Least Squares (FGLS) methods (Arbia, 2014).

When $\lambda = 0$ and $\rho \neq 0$, the Model becomes

$$y = Z\beta + u \quad (7)$$

$$u = \rho Wu + \mathcal{E} \quad |\rho| < 1 \quad (8)$$

with the regressors Z and the weights W non-stochastic. This model is referred to in the literature as the Spatial Error Model (SEM) (Anselin, 1988; Arbia, 2006; LeSage and Pace, 2009).

If $\mathcal{E} | X \approx i.i.d. N(0, \delta_{\varepsilon}^2 I_n)$, then we have that $u = (I - \rho W)^{-1} \mathcal{E}$ as in (5), so we can write:

$$E(u) = 0$$

$$E(uu^T) = \sigma_{\mathcal{E}}^2 (I - \rho W)^{-1} (I - \rho W^T)^{-1} = \sigma_{\mathcal{E}}^2 \Omega \quad (9)$$

A formulation that considers both heteroscedastic and autocorrelated error terms. In these circumstances the GLS procedure may be applied only if the value of the parameter ρ is known *a priori*, a circumstance which happens only very rarely in empirical cases. Notice that from (8), we have

$$(I - \rho W)u = \mathcal{E} \quad (10)$$

and models (1) and (2) can thus also be written as:

$$\begin{aligned} (I - W)y &= (I - \rho W)Z\beta + (I - \rho W)u \\ y &= \rho Wy + Z\beta - WZ\beta + \mathcal{E} \\ y &= \rho Wy + Z\beta - WZy + \mathcal{E} \end{aligned} \quad (11)$$

With $\lambda = \rho \beta$ and one may think of estimating model (10) directly. However, two problems emerge. First of all, (11) is over-parameterized due to the restriction

$\gamma = \rho \beta$. Secondly, the term Wy is correlated with the error term, thus producing endogeneity. let us consider that, from (11):

$$(I - \rho W)y = Z\beta - WZy + \mathcal{E}$$

and so,

$$\begin{aligned} E[(Wy) \mathcal{E}T] &= E[W(I - \rho W)^{-1}(Z\beta - WZy)] \\ &+ W(I - \rho W)^{-1}E[\mathcal{E}T] = W(I - \rho W)^{-1}(Z\beta - WZy)E(\mathcal{E}T) \\ &+ W(I - \rho W)^{-1}E[\mathcal{E} \mathcal{E}T] = \sigma^2 W(I - \rho W)^{-1} \quad I \neq 0 \end{aligned} \quad (12)$$

So the error is endogeneous, in that it is correlated with the spatially lagged variable Wy . As a consequence of the endogeneity of the errors, the OLS procedure loses its optimal properties.

In principle, an instrumental variable procedure could have been adopted to accommodate endogeneity. However, Kelejian and Prucha (1998) proved that such a procedure is not consistent due to the fact that it is not possible to identify instruments for Wy which are linearly independent of the other two regressors, Z and WZ .

A feasible GLS procedure (FGLS) can be obtained along the following steps (Kelejian and Prucha, 1998):

- Step 1: first of all obtain a consistent estimate β say $\hat{\beta}$
- Step 2: use these estimates to obtain an estimate of u say $\hat{u}$
- Step 3: use $\hat{u}$ to estimate ρ in (8), say $\hat{\rho}$
- Step 4: use $\hat{\rho}$ to transform model (7) as

$$(I - \hat{\rho}W)y = (I - \hat{\rho}W)Z\beta + \varepsilon$$

- Step 5: finally, since the transformed model now contains stochastic disturbances which satisfy the requisites, estimate b via OLS on the transformed data corresponding to the GLS procedure.

2.1.3 The Complete SARAR (1,1) Model

In this study, SARAR(1,1) model will be estimated using Maximum Likelihood (ML) and the Generalized Spatial Two-Stage Least Squares (GS2SLS) (Arbia, 2014)

Let us consider the case where, in (1) and (2), we set $\beta = 0$. We have:

$$y = \lambda Wy + u \quad |\lambda| < 1 \quad (13)$$

$$u = \rho Wu + \mathcal{E} \quad |\rho| < 1 \quad (14)$$

(Arbia, 2014)

we thus have:

$$(I - \lambda W)y = u \quad y = (I - \lambda W)^{-1}u \quad (15)$$

and

$$(I - \rho W)u = \mathcal{E} \quad u = (I - \rho W)^{-1}\mathcal{E} \quad (16)$$

Combining (12) and (13) we have:

$$y = (I - \lambda W)^{-1}(I - \rho W)^{-1}\mathcal{E} \quad (17)$$

$$\begin{aligned} E(yy^T) &= E[(I - \lambda W)^{-1}(I - \rho W)^{-1}\mathcal{E}\mathcal{E}^T(I - \lambda W)^{-T}(I - \rho W)^{-T}] \\ &= \sigma_{\mathcal{E}}^2(I - \lambda W)^{-1}(I - \rho W)^{-1}(I - \lambda W)^{-T}(I - \rho W)^{-T} \\ &= \sigma_{\mathcal{E}}^2 \end{aligned} \quad (18)$$

so that the inverse of Ω is now:

$$\begin{aligned} \Omega^{-1} &= (I - \lambda W^T)(I - \rho W^T)(I - \rho W)(I - \lambda W) \\ &= [I - (\lambda + \rho)W^T + \lambda\rho W^T W^T] * [I - (\lambda + \rho)W + \lambda\rho W W^T] \end{aligned} \quad (19)$$

where the two parameters λ and ρ are present in the form of a sum and of a product and so they cannot be identified univocally. This fact has been considered in the literature to suggest that a complete model of the kind reported in Equations (13) and (14) is not feasible in practice. However, Kelejian and Prucha (1998) proved that this only happens when $\beta = 0$ and it is not the case conversely when $\beta \neq 0$, which is what usually happens in the generality of cases of interests in spatial econometrics. In this case we can define a more general spatial model which encompasses the Spatial Lag and the Spatial Error models previously discussed above. This model, as already said, was termed a SARAR(1,1) model by Kelejian and Prucha (1998), but is also referred to in the literature as the *General Spatial Model* by Anselin (1988) or as an *SAC* model by LeSage and Kelly (2009).

2.1.4 The Generalized Spatial Two-Stage Least Squares (GS2SLS)

The Generalized Spatial Two-Stage Least Squares (GS2SLS) was introduced by Kelejian and Prucha (1998) and accounts for both the problem of endogeneity of Wy and the problem of spatial correlation among the stochastic disturbances. It is an extension of the 2SLS methodology and combination with the GMM estimator to account for the spatial correlation structure in the disturbances. The GS2SLS procedure can be obtained using the following steps:

<http://www.bjs-uniben.org/>

Let us consider again the full model contained in (1) and (2), we thus have as earlier stated above

$$y = \lambda W y + X \beta_{(1)} + W X \beta_{(2)} + u \quad |\lambda| < 1$$

$$u = \rho W u + \varepsilon \quad |\rho| < 1$$

$$\mathcal{E} | X \approx i.i.d. N(0, \sigma_{\varepsilon n}^2 I_n).$$

Step 1: first of all obtain a consistent estimate of the parameters β and λ , say $\hat{\beta}$ and $\hat{\lambda}$

Step 2: use these estimates to obtain an estimate of u in (1), say $\hat{u}$

Step 3: use $\hat{u}$ to estimate ρ in (2), say $\hat{\rho}$

Step 4: use $\hat{\rho}$ to transform model (2) as

$$(I - \hat{\rho}W)y = (I - \hat{\rho}W)Z\beta + \varepsilon$$

Step 5: finally, estimate the parameters of such a transformed model using 2SLS with the transformed variables

$$Z^* = (I - \hat{\rho}W)Z; WZ^* = W(I - \hat{\rho}W)Z; \text{ and } W^2Z^* = W^2(I - \hat{\rho}W)Z$$

2.2 Interpretation of the Parameters in Spatial Econometric Models

In a standard linear regression model the regression parameters have an easy interpretation in that they represent the partial derivative of the dependent variable y with respect to the independent variables:

$$b_i = \frac{\partial y_i}{\partial X_i} \quad (20)$$

which can therefore be straightforwardly interpreted as the variation induced on variable y of a unitary increase in the single independent variable X_i .

However, in the spatial econometric models interpretation of the parameters is less immediate and requires some clarification. In fact, a variation of variable X observed in location i does not only have an effect on the value of variable y in the same location, but also on variable y observed in other locations.

The impact of each variable X on y can then be described through the partial derivatives $\frac{\partial E(y)}{\partial X}$ which can be arranged in the following matrix:

$$\frac{\partial E(y)}{\partial X} = S = \begin{bmatrix} \frac{\partial E(y_1)}{\partial X_1} & \dots & \frac{\partial E(y_i)}{\partial X_n} \\ \dots & \dots & \dots \\ \frac{\partial E(y_n)}{\partial X_1} & \dots & \frac{\partial E(y_n)}{\partial X_n} \end{bmatrix} \quad (21)$$

whose single entry is defined as:

$$\text{http://www.bjs-uniben.org/}$$

$$s_{ij} = \frac{\partial E(y_i)}{\partial X_j} \quad (22)$$

Based on Lasage and Pace (2009) who suggested and employed the following impact measures that can be calculated for each of the independent variables X_i included in the model:

- (1) **Average Direct Impact (ADI)**. This measure refers to the average total impact of a change in X_I on y_i for each observation, which implies the average of all diagonal entries in matrix S :

$$ADI = n^{-1}tr(S) = \sum_{i=1}^n \frac{\partial E(y_i)}{\partial X_i} \quad (23)$$

- (2) **The Average Total Impact to an observation (ATIT)** which is the measure of the impact produced on a single observation by all the other observation is calculated as the sum of the i th row matrix S :

$$ATIT_i = n^{-1} \sum_{i=1}^n s_{ij} = n^{-1} \sum_{i=1}^n \frac{\partial E(y_i)}{\partial X_i} \quad (24)$$

- (3) A measure related to the impact produced by one single observation on all other observations, termed **Average Total Impact From (ATIF) an observation**. For each observation this is calculated as the sum of the j -th column of matrix S :

$$ATIF_i = n^{-1} \sum_{j=1}^n s_{ij} = n^{-1} \sum_{j=1}^n \frac{\partial E(y_i)}{\partial X_j} \quad (25)$$

- (4) A global measure of the average impact obtained from the two preceding measures:

$$ATI = n^{-1} \sum_i^T S_i = n^{-1} \sum_{j=i}^n ATIT_i = n^{-1} \sum_{j=i}^n ATIF_i \quad (26)$$

Which is the Average of all of matrix S .

- (5) A measure of the **Average Indirect Impact (AII)** obtained as the difference between ATI and ADI:

$$AII = ATI - ADI \quad (27)$$

Which is the average of all the Off – diagonal entries of matrix S.

3. Results and Discussion

The data analysis in this study was implemented in R. This section begins with the estimation of the OLS estimation of Phillips Curve

Table 2: OLS Estimation of Phillips Curve

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.6985	0.0268	212.490	<2e-16 ***
log(Unemp2018)	-0.0019	0.0086	-0.223	0.825
F-statistic: 0.0498 on 1 and 35 DF, p-value: 0.8248				
Durbin-Watson test: DW = 1.5468, p-value = 0.0667				
studentized Breusch-Pagan test BP = 0.7520, df = 1, p-value = 0.3858				
Jarque Bera Test: X-squared = 64.979, df = 2, p-value = 7.772e-15				

In table 2, the result from OLS shows that there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0019 unit decrease in inflation rate in Nigeria which agrees with Philips curve hypothesis. This implies that when unemployment rate increase it will lead to decrease in inflation while decrease in unemployment rate would lead to increase in inflation rate. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017). Lastly, there is presence of autocorrelation at 10% level of significance while the error terms is not normally distributed ($p < 0.05$).

Table 3: Global Moran I for regression residuals

data:		
model: lm(formula = log(CPI2018) ~ log(Unemp2018), data = PC1)		
weights: W1		
Moran I statistic standard deviate = 1.5704, p-value = 0.05816		
alternative hypothesis: greater		
sample estimates:		
Observed Moran I	Expectation	Variance
0.12029787	-0.04148979	0.01061387

After this estimation of the OLS estimation of the Phillips Curve, the Moran I statistic help to investiage the presence of spatial autocorrelation since we are using cross sectional data with spatial characteristics. In table 2 above, the Moran I statistic for spatial autocorrelation test is only significant at 10%. Which signifies the presence of spatial autocorrelation in the estimated OLS model. The Moran I test for presence of spatial autocorrelation is necessary to support the appropriateness of the application of spatial models.

Table 3: Monte-Carlo simulation of Moran I

```

data: log(PC1$Unemp2018)
weights: W1
number of simulations + 1: 10001

statistic = 0.45214, observed rank = 10001, p-value = 9.999e-05
alternative hypothesis: greater

```

In table 3 above, the Monte-Carlo simulation of Moran I statistic at 10,000 simulations revealed the presence of spatial autocorrelation at 1% level of significant. In application, the Monte-Carlo simulation of Moran I statistic is preferred as being robust (Arbia, 2006). This further suggested the presence of spatial autocorrelation in the estimated OLS model.

Table 4: Spatial Lag Model for Philips Curve using Maximum Likelihood Estimation

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.4108	1.2294	3.5876	0.0003
log(Unemp2018)	-0.0011	0.0082	-0.1292	0.8972
Rho: 0.22576, LR test value: 1.0358, p-value: 0.30879				
Asymptotic standard error: 0.2159, z-value: 1.0459, p-value: 0.2956				
LM test value: 0.2758, p-value: 0.5995				
ADI: -0.0011				
AII: -0.0003				
ATI: -0.0014				

In table 4, there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0011 unit decrease in inflation rate in Nigeria which agrees with Philips curve hypothesis. This implies that when unemployment rate increase it will lead to decrease in inflation while decrease in unemployment rate would lead to increase in inflation rate. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017).

The result from Spatial Lag Model shows a unit increase in unemployment leads to a decrease of 0.0011 unit in inflation rate in Nigeria using the Average Direct Impact (ADI). In addition, a unit increase of unemployment in one state of Nigeria produces a total impact of reduction of 0.0014 in inflation rate using the Average Total Impact (ATI). This result agrees with the work of Iyeli and Ekpung (2017) but contradicts the work of Orji *et al.* (2015).

Table 5: Spatial Lag Mixed Model for Philips Curve using Maximum Likelihood Estimation

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.4194	1.2315	3.5887	0.0003
log(Unemp2018)	-0.0005	0.0110	-0.0485	0.9613
lag.log(Unemp2018)	-0.0012	0.0163	-0.0713	0.9432
Rho: 0.2246, LR test value: 1.0192, p-value: 0.31272				
Asymptotic standard error: 0.21601, z-value: 1.0397, p-value: 0.29846				
LM test value: 0.39866, p-value: 0.52778				
ADI: -0.0006				
AII: -0.0016				
ATI: -0.0022				

In table 5, there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0005 unit decrease in inflation rate in Nigeria while increase in unemployment at lag 1 (previous year) leads to 0.0012 unit decrease inflation rate in Nigeria which agrees with Philips curve hypothesis. This implies that when unemployment rate increase it will lead to decrease in inflation while decrease in unemployment rate would lead to increase in inflation rate. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017).

The result from Spatial Lag mixed Model shows a unit increase in unemployment leads to a decrease of 0.0006 unit in inflation rate in Nigeria using the Average Direct Impact (ADI). In addition, a unit increase of unemployment in one state of Nigeria produces a total impact of reduction of 0.0022 in inflation rate using the Average Total Impact (ATI). This result agrees with the work of Iyeli and Ekpung (2017) but contradicts the work of Orji *et al.* (2015).

Table 6: Spatial Error Model for Phillips curve using Maximum Likelihood Estimation

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.6960143	0.0283356	201.0201	<2e-16
log(Unemp2018)	-0.0010782	0.0090423	-0.1192	0.9051
Lambda: 0.22594, LR test value: 1.0334, p-value: 0.30936				
Asymptotic standard error: 0.21586, z-value: 1.0467, p-value: 0.29524				

In table 6, the spatial Error Model using Maximum Likelihood method revealed that there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0011 unit decrease in inflation rate in Nigeria. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017).

Table 7: Spatial Error Model for Phillips curve using Feasible Generalized least squares Estimation

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.6961714	0.0281859	202.0932	<2e-16
log(Unemp2018)	-0.0011308	0.0089966	-0.1257	0.9
Lambda: 0.21202 (standard error): 0.50478 (z-value): 0.42002				
Residual variance (sigma squared): 0.00024491, (sigma: 0.01565)				

In table 7, the spatial Error Model using Feasible Generalized Least squares Estimation revealed that there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0011 unit decrease in inflation rate in Nigeria. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017).

Table 8: SARAR Model for Phillips using Maximum Likelihood Estimation

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.4107867	1.2294401	3.5876	0.0003337
log(Unemp2018)	-0.0010572	0.0081836	-0.1292	0.8972157
Rho: 0.22576, LR test value: 1.0358, p-value: 0.30879				
Asymptotic standard error: 0.21585				
z-value: 1.0459, p-value: 0.29561				
LM test value: 0.27577, p-value: 0.59949				
ADI: -0.0011				
AII: -0.0003				
ATI: -0.0014				

In table 8, there is a negative relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0011 unit decrease in inflation rate in Nigeria which agrees with Philips curve hypothesis. This implies that when unemployment rate increase it will lead to decrease in inflation while decrease in unemployment rate would lead to increase in inflation rate. This result agrees with Orji *et al.* (2015), but contradicts the work of Iyeli and Edeme (2017).

The result from SARAR Model shows a unit increase in unemployment leads to a decrease of 0.0011 unit in inflation rate in Nigeria using the Average Direct Impact (ADI). In addition, a unit increase of unemployment in one state of Nigeria produces a total impact of reduction of 0.0014 in inflation rate using the Average Total Impact (ATI). This result agrees with the work of Iyeli and Ekpung (2017) but contradicts the work of Orji *et al.* (2015).

Table 9: SARAR Models for Phillips using Generalized Spatial Two-Stage Least Squares (GS2SLS)

Dependent Variable: Log(CPI2018)

Coefficients	Estimate	Std. Error	t value	Pr(> t)
Rho	2.3625413	3.0013969	0.7871	0.4312
(Intercept)	-7.7776370	17.1203057	-0.4543	0.6496
log(Unemp2018)	0.0070538	0.0170768	0.4131	0.6796
Residual variance (sigma squared): 0.00059083, (sigma: 0.024307)				

In table 9, SARAR model using Generalized Spatial Two-Stage Least Squares (GS2SLS) revealed that there is a positive relation between unemployment and Inflation in Nigeria. One unit increase in unemployment leads to 0.0071 unit increase in inflation rate in Nigeria which disagrees with Philips curve hypothesis. This implies that when unemployment rate increase it will lead to increase in inflation while decrease in unemployment rate would lead to decrease in inflation rate. This result agrees with Iyeli and Edeme (2017) but contradict Orji *et al.* (2015).

4. Conclusion

Spatial econometrics should be viewed in a wide sense involving developments of models and statistical tools for the analysis of externalities, spill overs, interactions etc., in various areas including economics, geography and regional science, etc. The result from OLS shows that there is a negative relation between unemployment rate and Inflation rate in Nigeria. One unit increase in Unemployment rate leads to decrease of 0.0019 unit in Inflation rate in Nigeria which agreed to Philips curve.

In addition, result from Spatial Lag Model shows a unit increase in Unemployment leads to decrease of 0.0011 of Inflation rate in Nigeria. In addition, a unit increase of Unemployment in one state of Nigeria produces a total impact of reduction of 0.0014 in Inflation rate. This result agrees with Philips curve.

References

- Abu, N. (2019). Inflation and Unemployment Trade-off: A Re-examination of the Philips Curve and its stability in Nigeria. *Contemporary Economics*, 13(1), 21-34.
- Anselin, L. (1982). A Note on Small sample properties of Estimators in a First-Order Spatial Autoregressive Model. *Environment and Planning A*, 14, 1023-1030
- Anselin, L. (1988). *Spatial Econometrics: Methods and Models*, Dordrecht: Kluwer Academic Publishers.
- Arbia, G. (2006). *Spatial Econometrics: Statistical Foundations and Applications to Regional Economic Growth*, Heidelberg: Springer-Verlag.
- Arbia, G. (2014). *A Primer for Spatial Econometrics with Application in R*. New York: Palgrave Macmillan
- Ayinde, R. A, Akanegbu, B. and Jelilov, G. (2021). Estimation of New Keynesian Phillips Curve for Nigeria. *The Journal of Developing Areas*, 55(3), 149-158.
- Baltagi, B. H. (2001). *Econometrics Analysis of Panel Data* (2nd ed). Chichester: John Wiley & Sons.

- Chuku, C, Attan, J. and Obioesio, F. (2017). Testing for the stability and Persistence of the Phillips Curve for Nigeria. *CBN Journal of Applied Statistics*, 8(1), 123-147.
- Daniel, S. U., Israel, V. C., Chidubem, C. B. and Quansah, J. (2021). Relationship between inflation and unemployment: Test Philips Curve Hypotheses and Investigating the causes of inflation and unemployment in Nigeria. *Traectoria Nauki=Path of Science*, 7(9), 1013 - 1027
- Dempsey, C. (2014). Tobler's First Law of Geography. *Geography basics*. <https://www.geographyrealm.com/toblers-first-law-geography/> retrieved on 09-09-2021.
- Dumlao, L. F. (2012). The Relationship between Dynamic Price and Dynamic Unemployment: The case of Malaysia, the Philippines, Singapore and Thailand. *Intl J. of Economic Perspectives*, 1-29 www.highbeam.com
- Edeme, R. K. (2018). Providing an empirical insight into Nigeria's non-acceleration rate of unemployment. *Journal of Development Policy and Practice*, 3(2), 179-190.
- Efayena, O. O. and Olele, H. E. (2020). A Validation of the Phillips Curve Hypothesis in Nigeria: A Quarterly data-based approach. *MPRA*, <https://mpra.ub.uni-muenchen.de/988804/> downloaded 16-03-2022
- Elhorst, J. P. (2010). *Applied Spatial Econometrics: Raising the Bar*, *Spatial Economic Analysis*, 5(1), 9-28, DOI: 10.1080/17421770903541772.
- Friedman, M. (1968). The role of monetary policy. *American Economic Review*, 57(1), 1 - 17.
- Furuoka, F. (2007). Does the Phillips curve really exist? New empirical evidence from Malaysia. *Economics Bulletin*, 5(16), 1-14.
- Gerschgorin, S. (1931). Über die Abgrenzung der Eigenwerte einer Matrix. *Izv. Akad. Nauk. USSR Otd. Fiz.-Mat.*, 7, 749-54.
- Haug, A. A. and King, I. (2014). In the long run, US unemployment follows inflation like a Faithful Dog. *Journal of Macroeconomics*, 41, 42-52.
- Islam, F., Hassan, K., Mustafa, M. and Rahman, M. (2003). The empirics of US Phillips curve: a revisit. *American Business Review*, 20(1), 107-112.
- Iyeli, I. I. and Ekpung, E. G. (2017). Price expectation and the Philips curve hypothesis: the Nigeria case. *International Journal of Development and Economic Sustainability*, 5(4), 1-10.
- Karahan, O., Colak, O. and Bolukbaşı, O. F. (2012). Tradeoff between Inflation and Unemployment in Turkey. *The Empirical Economics Letters*, 11(9), 973-980.
- Katria, S., Bhutto, N. A., Butt, F., Domki, A. A., Khawaja, H. A. and Khalid, J. (2011). Trade off between inflation and unemployment. *International Conference on Business Management*, 1-18.
- Kelejian, H. H. and Prucha, I. R. (1998). A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances. *The Journal of Real Estate Finance and Economics*, 17, 99-121.
- Kogid, M., Asid, R., Mulok, D., Lily, J. and Loganathan, N. (2011). Inflation-Unemployment Trade-off Relationship in Malaysia. *Asia Journal of Business and management Sciences*, 1(1), 100-108.
- LeSage, J. and Pace, K. (2009). *Introduction to Spatial Econometrics*, Boca Raton, FL: Wiley.
- Li, T. J. J., Sen, S. and Hecht, B. (2014). Leveraging Advances in Natural Language Processing to better Understand Tobler's First Law of Geography. *SIGSPATIAL '14: Processing of the 22nd ACM SIGSPATIAL Intl Conf. on Advances in Geographic Information Systems*, 513-516.
- Lucas, R.E. (1976). *Econometric policy evaluation: a critique*. *Carnegie-Rochester Conference Series on Public Policy*, 1, 19-46.
- Odo, S. I., Elom-Obed, O. F., Nwachukwu, O. J. and Okoro, O. T. (2017). Understanding the relationship between unemployment and inflation in Nigeria. *Advances in Research*, 9(2), 1-12.
- Okafor, I. G., Ezeaku, H. C. and Ugwuebe, S. U. (2016). Responsiveness of unemployment to inflation: empirical evidence from Nigeria. *International Journal of Scientific Research in Science and Technology*, 2(4), 173-179.
- Okoro-Ugochukwu N. A. and Adenomon M. O. (2021). Application Of Okun's Law On

- Nigerian Economy: Evidence From Spatial Econometrics Analysis. *Science World Journal*, 16(3), 291-295 www.scienceworldjournal.org
- Orji, A. Anthony-Orji, O. I. and Okafor, J. C. (2015). Inflation and unemployment nexus in Nigeria: A test of Philips curve. *Asian Economic and Financial Review*, 5(5), 283-299.
- Paelinck, J.H.P. and Klaassen, L.H. (1979). *Spatial Econometrics*, Aldershot, UK: Gower.
- Phelps, E. (1967). Phillips curve, expectation of inflation, and optimal inflation over time. *Economica*, 34, 254–281.
- Saad, B. and Salim, M. I. A. (2017). Inflation and unemployment in Nigeria: an ARDL approach. *World Journal of Economic and Finance*, 3(2), 069-074.
- Samuelson, P.A. and Solow, R.M. (1960). Analytical aspect of anti-inflation policy, *American Economic Review*, 50(2), 177–194.
- Toda, H. Y. and Yamamoto, T. (1995). Statistical Inference in Vector Autoregressions with Possible Integrated Processes. *Journal of Econometrics*, 66, 225-250.
- Uche, E. (2019). Inflation and Unemployment Dynamics in Nigeria: A Re-examination of the Philips Curve Theory. *International Journal of Scientific and Research Publication*, 9(1), 876-884.
- Umoru, D. and Anyiwe, M. A. (2013). Dynamics of inflation and unemployment in a Vector Error Correction Model. *Research on Humanities and Social Sciences*, 3(3), 20-29.
- Whittle, P. (1954). On Stationary Processes in the Plane. *Biometrika*, 41, 434–49.

Appendix**Table 1:** Cross Sectional Data on Unemployment rate and Consumer Price Index (CPI) for 2018

States	Unemployment rate (%)	CPI
Sokoto	26	291.6
Zamfara	18	297.4
Katsina	14.3	294.4
Jigawa	26.5	293.3
Yobe	29	300.4
Borno	31.4	293.1
Adamawa	20.8	295.5
Gombe	27	291.9
Bauchi	23.5	278.2
Kano	31.3	297
Kaduna	26.8	301.5
Kebbi	20.1	296.4
Niger	20.9	293.7
FCT	24.4	295.2
Nassarawa	27.4	301.1
Plateau	29.8	290.9
Taraba	19	296
Benue	20.1	298.9
Kogi	19.7	299
Kwara	21.1	299.2
Oyo	10.3	297
Osun	10.1	299.3
Ekiti	20.2	293.9
Ondo	14.2	298.3
Edo	25.1	295.7
Anambra	17.5	299.2
Enugu	18.7	297.2
Ebonyi	21.1	296.5
Cross River	30.6	295.2
Akwa Ibom	37.7	296.3
Abia	31.6	297.2
Imo	28.2	296.3
Rivers	36.4	300.7
Bayelsa	32.6	310.1
Delta	25.4	300.7
Lagos	14.6	302.6
Ogun	16.4	296.8

Source: NBS (2018)

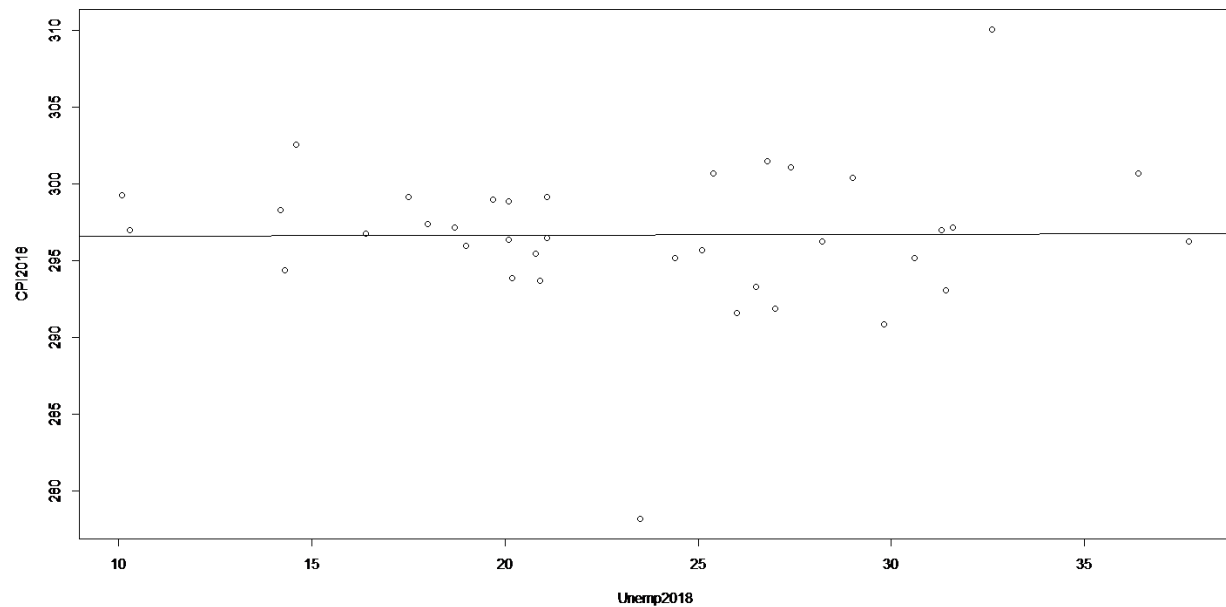


Fig 1: Scattered plot between Inflation and Unemployment in all the states in Nigeria in 2017 to 2018.

Fig 1 above presents a scatter plot of cross section of consumer price index against unemployment rate of states in Nigeria in 2018. This is done to show the possible relationship that exist between consumer price index and unemployment rate among the states in Nigeria. The Fig 1 shows a possible negative relationship between consumer price index and unemployment rate among states in Nigeria.