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Modeling and Forecasting Exchange Rates Volatility Using Selected GARCH Models

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Abstract. Modeling and forecasting foreign exchange rate volatility is a critical issue in various fields of finance today. Volatility of the Nigeria foreign exchange market was studied using the generalized autoregressive conditional heteroskedasticity (GARCH), exponential GARCH (EGARCH), threshold GARCH (TGARCH) and integrated GARCH (IGARCH) models. In the study, the exchange rates of Nigeria Naira and four other country's currency were compared. Based on daily data of the exchange rates of Nigeria Naira and respective US Dollar (USD), EURO (EUR), Pound Sterling (PDS) and the Japanese Yen (YEN), the tests for asymmetry and normality were evaluated using skewness, kurtosis and Jarque-Bera statistics. It was observed that the data confirm non-normality and asymmetry. This is evidence from the descriptive statistics including readings from the skewness, kurtosis and Jarque-Bera statistics. All the exchange rates were statistically significant at 5% level of significance except for the USD which is statistically insignificant. Also, in all the selected GARCH models, the AC, PAC and Q-Statistics show no presence of autocorrelation while volatility of the returns series and leverage effect are positively correlated at 5% confidence level except for the DYEN return series which is negative and significant. The model performance is assessed by checking the lowest Akaike Information Criterion (AIC) and it can be deduced that the IGARCH model is preferred for estimating daily return series for USD, PDS and EUR whereas TGARCH is seen to be appropriate for YEN during the study period.

Keywords: : Exchange Rates, Time Series, Modeling, Volatility, GARCH Models, Forecasting.

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1. Introduction

Exchange rate volatility is a major challenge for a developing economy like Nigeria which is highly dependent on foreign trade. The exchange rates become extremely volatile and there is no limit to its volatility with time which has resulted in import-dependent production structure, weak non-oil export earnings,

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inadequate foreign capital inflow, excess demand for foreign exchange relative to supply, heavy debt burden, poor standard of living, etc. Engle (1982) proposed autoregressive conditional heteroscedasticity (ARCH (q)) model that seemed to capture the empirical characteristics in the financial time series. The ARCH model has non-constant variance conditioned on the past which is a linear aggregate of recent past disturbances. The generalized autoregressive conditional heteroscedasticity (GARCH) model and its extensions (EGARCH, TGARCH, IGARCH, FIGARCH, etc.) have been also proposed to capture both the symmetric and asymmetric volatilities in stock markets, inflation and exchange rates. Using daily and monthly data sets of Pakistani stock market in 1990, Rizwan and Khan (2007) discovered evidence of volatility clustering and persistence. Also, Selcuk *et al.* (2004) examined unpredictability in arising securities exchanges and tracked down instability persistency. The parameter estimate of the GARCH model for the Ghana stock exchange was found to be close to unity by Magnus and Fosu (2006). Bala and Asemota (2013) used monthly exchange rate return series to examine the volatility of exchange rates using GARCH model. Their study compared estimates from different GARCH models with break in relation to US dollar rates with break points that were set by the environment. The results showed that estimates of volatility models with breaks were better than estimates of GARCH models without breaks and that most models' persistence was lower when volatility breaks were added. Ajayi *et al.* (2019) examined the GARCH-type models (EGARCH, TGARCH, GJRGARCH, etc) with Nigeria foreign exchange returns and concluded that GJR-GARCH (1,1) is the best fitted model for all the currencies considered. Onyeka-Ubaka and Abass (2018) also examined aspects of economic and financial risks and their effects on volatility forecasting applied to selected banks' daily stocks. They concluded that the generalised student t distribution for the Bilinear GARCH (BL-GARCH) model captured heavier and lighter tails in high frequency financial time series data and act as general distribution for empirical characteristics like volatility clustering, leptokurtosis and the leverage effect between returns and conditional variances. Many other researchers who worked on exchange rates volatility, GARCH-type models and forecasting exchange rates dynamics include: Taylor (1987), Longmore and Robinson (2004), Kandil and Mirzaie (2008), Kasman *et al.* (2011), Kalli and Griffin (2015), Olowofeso *et al.* (2015), Humala and Rodriguez (2010), Zivot (2009), Hu and Tsay (2014), Barunik *et al.* (2016), Epaphra (2017) and Ogundeji *et al.* (2021).
This study considers Nigeria Naira exchange rates with respect to four currencies (the US Dollar, Pounds Sterling, Euro and Japanese Yen). The volatilities of these exchange rates were studied using selected GARCH type models (based on the performance of the GARCH type models in previous researches). This is to investigate characteristics of exchange rate volatility in Nigeria and to understand empirical facts about leverage effect, volatility clustering and persistence. This study is to establish whether the daily returns exhibit signs of asymmetry (skewness) or are normally distributed; to fit the best model, compute the statistical significance of parameter estimates of the Nigeria exchange rate index and to ascertain the performance of the fitted models.

2. The Models

2.1 *ARCH (q) Model*

Problems pertaining to the volatility of economic variables could be empirically investigated in a rigorous manner using the selected models. The conditional variance, σ_t^2, is a linear function of the lagged squared residuals, e_t. The residuals, e_t from the conditional mean equation is obtained through ordinary least squares regression. For an ARMA (1, 1) model, the conditional mean equation will be:

$$r_t = \emptyset_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{1}$$

In addition, the squared residuals, e_t^2 is regressed on a constant and q lags as:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_q e_{t-q}^2 \tag{2}$$

The null hypothesis: $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$; states that there is no ARCH effect up to order q against the alternative:
$H_1 : \alpha_i > 0$; for at least one i = 1, 2, . . . , q.
This procedure examines the residuals of the exchange rate returns series for evidence of heteroscedasticity using the Lagrange multiplier (LM) test of Engle (1982). Finally, the test statistic for the joint significance of the q-lagged squared residuals is the number of observation times the R-squared (TR^2) from the regression, where TR^2 is evaluated against χ^2 (q) distribution. The ARCH (q) model proposed by Engle (1982) has some weaknesses like:
(i) Because the square of the shocks that came before it is taken into account, the model assumes that both positive and negative shocks have the same impact on volatility.
(ii) In order for an ARCH(1) model's α_1^2 to have a finite fourth moment, it must fall within the range [0, 1/3]. Higher-order ARCH models make the constraint more difficult to understand.
(iii) The ARCH model does not tell us anything new about how financial time series variations come about. They only provide a mechanical means of describing the conditional variance's behaviour. It does not say anything about what causes such behaviour.
(iv) Because they respond slowly to significant isolated shocks to the return series, ARCH models are likely to over predict volatility.
The principle of parsimony is applied in this study. This means that, in actual out-of-sample forecasts, simpler models almost always perform better than more complex models. As an initial regression, the null hypothesis is tested that there are no ARCH effects in the residual series using an autoregressive moving average (ARMA) model for the conditional mean in the returns series.

2.2 *Generalized Autoregressive Conditional Heteroskedastic (GARCH) Model*

Despite its simplicity, the ARCH (q) model frequently requires a large number of parameters to accurately describe an asset return's volatility process. Bollerslev (1986) centres around stretching out the curve model to take into consideration a more adaptable slack design. In addition to lags of the squared error term (e^2_{t-1},e^2_{t-2}, $\ldots$, e^2_{t-q}). The study introduces a conditional heteroskedasticity model that uses lags of the conditional variance as regressors in the model for the conditional variance: the summed up curve GARCH model. In order to investigate volatility clustering and persistence, the generalized autoregressive conditional heteroscedastic (GARCH) model is utilized. The current conditional variance (volatility) can be influenced by an infinite number of squared errors in the model's three parameters. By allowing the current conditional variance to depend on the first *p* past conditional variances as well as the *q* past squared innovations, the general framework of this model, GARCH (p, q), can be expressed as:

$$\sigma_t^2 = \omega \ + \ \sum_{j=1}^{q} \alpha_j \varepsilon_{t-1}^2 + \ \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2 \tag{3}$$

where *p* is the number of lagged σ^2 terms and *q* is the number of lagged ε^2 terms. For this study, the GARCH (1, 1) is used and represented as:

$$r_t = \ \mu + \ \varepsilon_t \tag{4}$$

$$\sigma_t^2 = \omega + \ \alpha_1 \varepsilon_{t-1}^2 + \ \beta_1 \sigma_{t-1}^2. \tag{5}$$

The mean and variance equations are, respectively, defined by (4) and (5), where, $\omega > 0$ and $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, r_t = return of the asset at time t, μ = average returns and ε_t = residual returns.

$$\varepsilon_t = \sigma_t \epsilon_t, \tag{6}$$

where ϵ_t is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1. The constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, are needed to ensure σ_t^2 is strictly positive (Poon and Granger, 2003). In this model, the mean equation is written as a function of constant with an error term. Since σ_t^2 is the one-period ahead forecast variance based on past information, it is called the conditional variance. The conditional variance equation specified is a function of three terms: (i) A constant term ω (ii) News about volatility from the previous period measured as the lag of the squared residuals from the mean equation: ε_{t-1}^2(the ARCH term); and (iii) Last period forecast variance: σ_{t-1}^2 (the GARCH term).

2.3 *Exponential Generalized Autoregressive Conditional Heteroskedastic (EGARCH) Models*

Since the lagged error terms in the equations for the conditional variance are squared, the ARCH and GARCH models do not account for the leverage effect. As a result, a positive error has the same effect on the conditional variance as a negative error. The EGARCH model, which Nelson (1991) proposed, is a model designed specifically to capture the asymmetry effect. Lagged error terms (rather than lagged squared errors) allow the natural logarithm of the conditional variance to change over time in an EGARCH model. The EGARCH (*p, q*) model for the conditional variance can be written as:

$$ln\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p} \beta_j ln\left(\sigma_{t-j}^2\right) + \sum_{i=1}^{q} \alpha_i \left\{\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - \sqrt{\frac{2}{\pi}}\right\} - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \tag{7}$$

The EGARCH model is asymmetric because the level $\frac{|\varepsilon_{t-i}|}{\sigma_{t-i}}$ is included with coefficient γ_i. Since the coefficient is typically negative, positive return shocks generate less volatility than negative return shocks assuming other factors remains unchanged. In order to capture asymmetric responses of the time-varying variance to shocks, this study employs EGARCH (1, 1) model, which has the following specification:

$$r_t = \mu + \varepsilon_t \tag{8}$$

$$ln\left(\sigma_t^2\right) = \omega + \beta_1 ln\left(\sigma_{t-1}^2\right) + \alpha_1 \left\{\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right\} - \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{9}$$

The mean and variance equations are, respectively, defined by (8) and (9).

2.4 *Integrated Generalized Autoregressive Conditional Heteroskedastic (IGARCH) Model*

The Integrated GARCH (p, q) models, both with or without trend proposed by Engle and Bollerslev (1986), are therefore part of a wider class of models with a property called "persistent variance" in which the current information remains important for the forecasts of the conditional variances for all horizon. So we have the Integrated GARCH (p, q) model with necessary condition:

$$\alpha\,(1) + \beta(1) = 1 \tag{10}$$

To consider the IGARCH (1, 1) which is characterized by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + (1 - \alpha_1)\sigma_{t-1}^2 \tag{11}$$

when $\omega = 0$, σ_t^2 is a martingale. Based on the nature of persistence in linear models, it seems that IGARCH(1,1) with $\omega > 0$ and $\omega = 0$ are analogous to random walks with and without drift respectively and are therefore natural models of "persistent" shocks. This turns out to be misleading, however: in IGARCH(1, 1) with $\omega = 0$, σ_t^2 collapses to zero almost surely and in IGARCH(1,1) with $\omega > 0$, σ_t^2 is strictly stationary and ergodic and therefore does not behave like a random walk, since random walks diverge almost surely.

2.5 *Threshold Generalized Autoregressive Conditional Heteroskedastic (TGARCH) Model*

TGARCH model of Zakoian (1990) has shown that positive and negative shocks of equal magnitude have a different impact on stock market volatility, which may be attributed to a leverage effect. The version TGARCH (1, 1) model specification of the conditional variance is given as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{12}$$

where d_{t-1} is a dummy variable, that is;

$$d_{t-1} = \begin{cases} 1 & if\ \varepsilon_{t-1} < 0,\ bad\ news \\ 0 & if\ \varepsilon_{t-1} > 0,\ good\ news \end{cases} \tag{13}$$

The coefficient γ is known as the symmetry or leverage term. When $\gamma = 0$, the model collapse to the standard GARCH forms, otherwise when the shock is positive (i.e. good news) the effect on volatility is α_1. But when it is negative (i.e. bad news) the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive; negative shocks have a larger effect on σ_t^2, than positive shocks (Lim and Sek, 2013).

3. Materials and Method

The first step in creating a volatility model for an asset return series in the GARCH model is to define a mean equation by determining whether the data is serially dependent. If they are, we build an econometric model (such as the ARMA or ARIMA) for the return series to eliminate any linear dependence. Afterward, the residuals of the mean equation are used to test for ARCH effects. If the ARCH effects are statistically significant, a volatility model is specified and then a joint estimation of the mean and volatility equations are performed (See more details at Caporale (2019), Yuehchao and Remya (2017)). The parameters of the models are estimated using maximum likelihood estimation (MLE). In

modeling volatility, choosing an appropriate model from various suitable models is essential. The model selection principle is a criterion to assess if the fitted model suggests an optimum balancing between parsimony and goodness of fit. This study utilized the frequently used model selection principle, the Akaike Information Criterion (AIC), to select the fitted models which satisfy the criteria such as the model has a low residual square error and low Bayesian information criterion; the coefficients of the model are significant statistically; the coefficients are invertible or stationary; the residuals are just background noise; the coefficients are overly correlated or redundant; the model is overly intricate and thrifty; a simpler model is effective; the model is appealing from a theoretical and intuitive standpoint: utilizing ACFs, PACFs, graphs, statistics, residuals etc. Hence, the best model has a lower AIC value and the highest log-likelihood metrics. For the GARCH family models, the method of maximization of log-likelihood is by taking partial derivatives of the log-likelihood with respect to each parameter $(\lambda, \alpha, \beta, \gamma)$, setting them to zero, and solving the resulting system of equations using any optimization algorithm.
Logarithmic returns were used to determine whether the weekly returns were normally distributed or showed signs of asymmetry (skewness). The computation formula for the weekly returns is given as (Abdullah *et al.,* 2017):

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{14}$$

where r_t is the returns of the currency in period *t*; P_t is the exchange rate of the currency in period *t* and P_{t-1} is the exchange rate of the currency in period *t* -1. The Augmented Dickey-Fuller (ADF) test was also employed to test the unit root hypothesis for each variable to determine the order in which the variables integrate based on the following regression:

$$\Delta y_t = \varphi + \beta_t + \alpha y_{t-1} + \sum_{i=1}^{k} d_i \Delta y_{t-1} + u_t \tag{15}$$

where u_t is a white noise error term and $\Delta y_{t-1} = y_{t-1} - y_{t-2}$, $\Delta y_{t-2} = y_{t-2} - y_{t-3}$, etc.
(15) tests the null hypothesis of a unit root against a trend stationary alternative. The Philips-Perron (PP) test is equally conducted on the return series to examine possible autocorrelation rather than the lagged variable method employed in the ADF test. The Philips–Perron test is computed from the equation below:

$$y_t = \delta_t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_p \Delta y_{t-p} + \mu_t \tag{16}$$

where δ_t may be 0, φ or $\varphi + \beta_t$. The Philips–Perron equation modifies the Dickey–Fuller test (Philips and Perron, 1988).

4. Results and Discussion

Time series data for this study obtained from the repository of the Central Bank of Nigeria online data bank (http://www.cbn.gov.ng) are the daily time series data which spans from January 2, 2018 to November 22, 2021. The series include Nigeria Naira exchange rates with respect to four currencies: the US Dollar (USD), Pounds Sterling (PDS), Euro (EUR) and Japanese Yen (YEN). The summary statistics of the time series data are presented in Table 1 which displays the variables' descriptive statistics for the period examined. It can be deduced from the variables' descriptive statistics that at 1% level of significance, the skewness, kurtosis, and Jarque Bera statistics demonstrates that all of the variables are non-normal or asymmetric. As a result, the time series plots of the four series are presented in Figures 1 to 4 below.

Table 1: Descriptive Statistics

Statistics	EURO (EUR)	Pounds Sterling (PDS)	US Dollar (USD)	Japanese Yen (YEN)
Mean	393.6242	448.1908	339.6920	3.123144
Median	370.9660	419.6955	307.0000	2.855700
Maximum	503.0075	583.3734	411.6300	3.777600
Minimum	329.5338	356.1507	305.5500	2.685400
Std. Dev.	53.98080	64.00870	40.54553	0.394643
Skewness	0.563795	0.726117	0.576038	0.418790
Kurtosis	1.695539	2.060354	1.653675	1.369450
Jarque-Bera	115.3306	116.0616	121.8008	130.3490
Sum	366464.1	417265.7	316253.2	2907.647
Sum Sq. Dev.	2709952.	3810316.	1528865.	144.8414

Time-varying volatility in the daily NGN/USD, NGN/PDS, NGN/EUR, and NGN/YEN exchange rates is empirically demonstrated as a non-stationary trend behaviour in Figures 1 to 4. From 2nd January 2018 to 22nd November 2021, the data indicates that the exchange rates for NGN/USD, NGN/PDS, NGN/EUR and NGN/YEN varied profusely. The highest rates were recorded for the NGN/USD, NGN/PDS, NGN/EUR and NGN/YEN exchanges between NGN/USD 411.03 and NGN/USD 411.63 in the time frame of 28th July 2021 to 24th August 2021 (CBN, 2021), NGN/PDS 583.3734 in the time frame of 3rd February 2021, NGN/EUR 501.17 and NGN/EUR 503.0075 in the time frame of 22nd March 2021 to 4th March 2021, and NGN/YEN 3.7024 and NGN/YEN 3.7776 in the time frame of 2nd March 20. Their low exchange rates ranged from NGN/USD 305.55 and NGN/USD 306.00 in the time period from January 3, 2018, to May 28, 2018, NGN/PDS 356.1507 and NGN/PDS 397.1319 in the period from January 28, 2019 to February 5, 2019, NGN/EUR 329.54 in the period from December 31, 2019 and NGN/YEN 2.685 in the period from September 24, 2018 to October 26, 2018, respectively. Additionally, the plots show that a few of the series USD, PDS, EUR and YEN have non-stationary behaviour. The time-series is formally tested for stationarity with the augmented Dickey Fuller test.

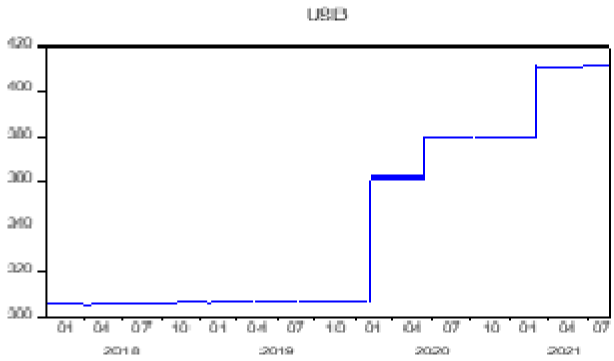

Figure 1: Time Series Plots of USD

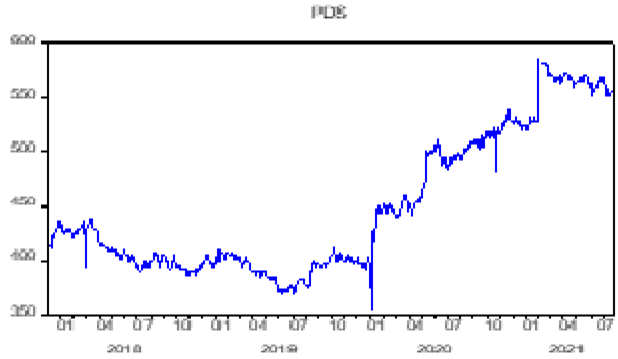

Figure 2: Time Series Plots of PDS

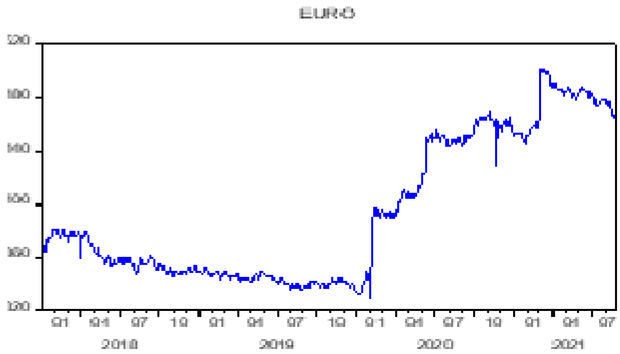

Figure 3: Time Series Plots of EUR

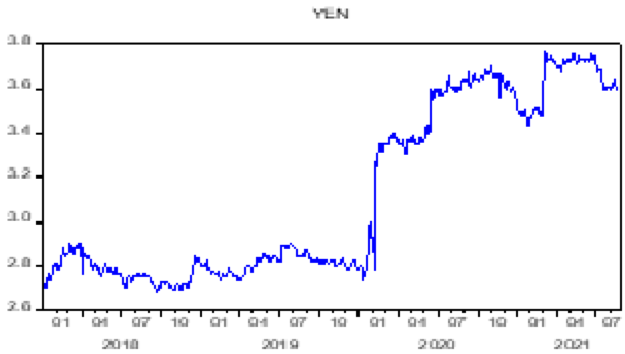

Figure 4: Time Series Plots of YEN

Table 2 displays the ADF test results for a linear time trend. At 5% level of significance, the unit root cannot be rejected using the ADF test with trend for four variables.

Table 2: Unit Root Test (ADF Test)

Variables	$\hat{k}$	Test Statistics	P-Value
USD	0	-2.864	0.9526
PDS	2	-2.865	0.9529
EUR	1	-2.864	0.8972
YEN	5	-2.8649	0.8642

Note: $\hat{k}$ is the AIC lag term used to select the optimal lag, to make the residuals white noise

From Table 2, since the absolute t-statistics values of all the series are below the absolute critical values of 2.864 (5% level of significance), or the p-values are greater than 5%; The unit root null hypothesis cannot be rejected for a sufficient number of reasons based on the selected data within the sample period. This suggests that the USD, PDS, EUR, and YEN are not fixed. As a result, we transform the data using the first difference in the series and conduct additional stationarity tests. It is clear from Figures 5 to 8 that each series has zero mean and approximately constant variance over time. This suggests that the series never moves. The ADF test buttresses the stationarity results. Hence, upon transformation by differencing the series, the variables USD, PDS, EUR and YEN are recoded as DUSD, DPDS, DEUR, and DYEN respectively as denoted subsequently ("D" denote first-difference).

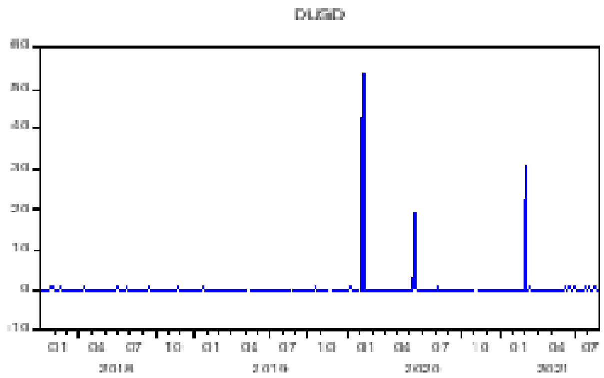

Figure 5: Time Series Plots of DUSD

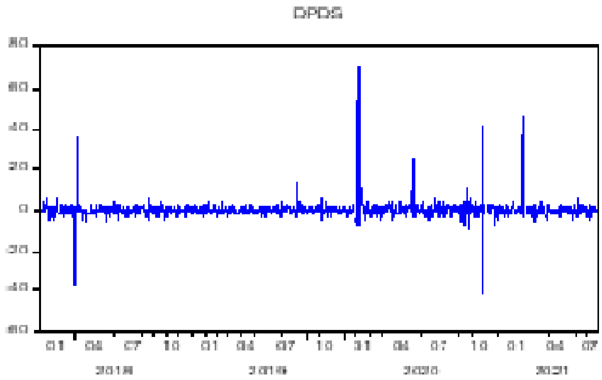

Figure 6: Time Series Plots of DPSD

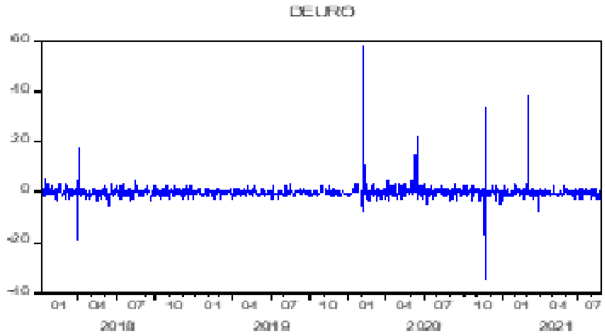

Figure 7: Time Series Plots of DEUR

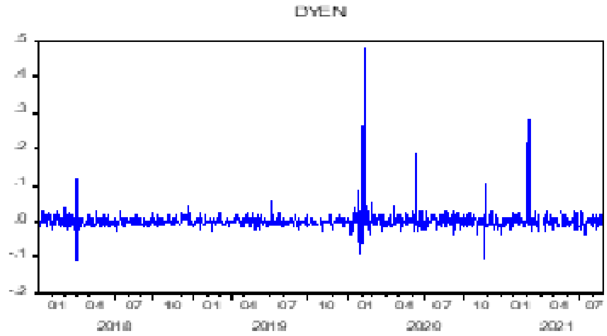

Figure 8: Time Series Plots of DYEN

Table 3: Unit Root Test (ADF Test)

Variables	$\hat{k}$	Test Statistics	P-Value
DUSD	0	-2.8644	0.0000
DPDS	1	-2.8644	0.0000
DEUR	0	-2.8644	0.0000
DYEN	4	-2.8644	0.0000

Note: $\hat{k}$ is the AIC lag term used to select the optimal lag, to make the residuals white noise

From Table 3, since all series have absolute t-statistics values greater than or equal to 2.8644 or p-values lower than 0.05 (5% level of significance); For DUSD, DPDS, DEUR, and DYEN, we have enough reasons to reject H0 at first difference. As a result, the DUSD, DPDS, DEUR, and DYEN series are stationary. Prior to estimating the ARCH and GARCH models, the study examines the exchange rate series to determine its statistical properties and to determine whether or not it satisfies the pre-requisites for the ARCH and GARCH models, which are clustering volatility and the ARCH effect in the residuals. The results of the test of clustering volatility in the residuals or error term are shown in Figures 9 to 12. The Figures show that huge and little mistakes happen in groups, which suggest that enormous returns are trailed by additional huge returns and little returns are also trailed by little returns.

In other words, the figures suggest that, while low exchange rates are likely to be followed by much lower exchange rates, periods of high exchange rates typically precede further periods of high exchange rates. ARCH and GARCH models can estimate this clustering volatility, which suggests conditionally heteroscedasticity for the residual or error term.

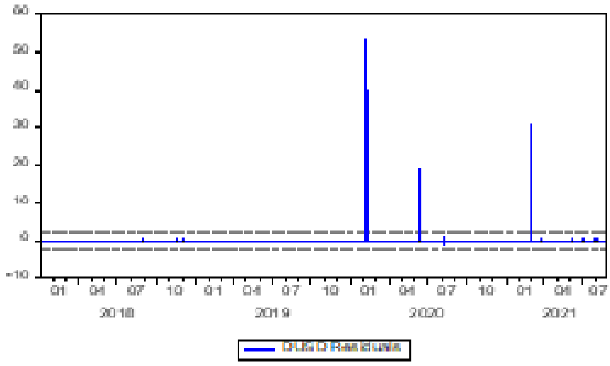

Figure 9: DUSD Residual Plot

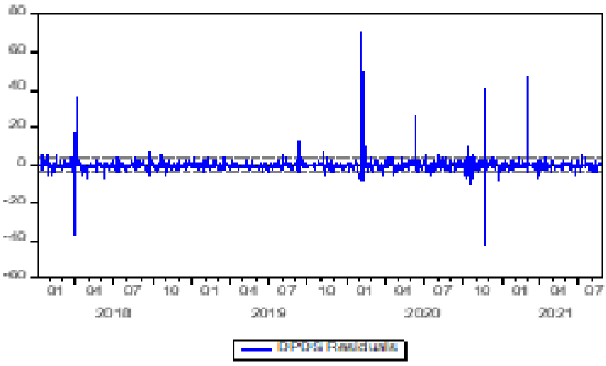

Figure 10: DPSD Residual Plot

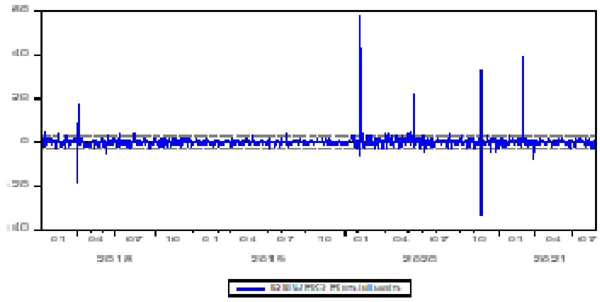

Figure 11: DEUR Residual Plot

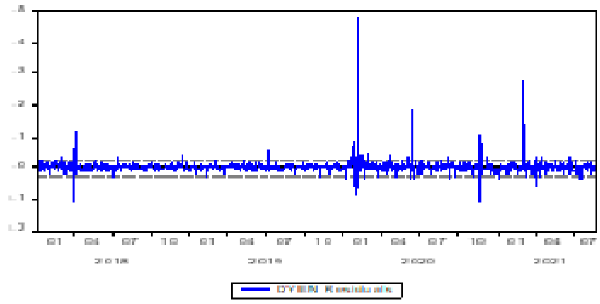

Figure 12: DYEN Residual Plot

The ARCH effect is concerned with a relationship that exists within the heteroscedasticity, sometimes referred to as serial correlation of the heteroscedasticity. When there is bunching in the variance or volatility of a particular variable, resulting in a pattern that is determined by some factor, it frequently becomes apparent (see Figures 9 to 12). The trade rates are further dependent upon heteroscedasticity test. The test results are shown in Table 4.

Table 4: Heteroscedasticity Test

Variables	χ^2	D.F	Prob > χ^2
DUSD	7.9533	1,947	0.0034
DPDS	17.9824	1,947	0.0005
DEUR	8.936	1,947	0.0065
DYEN	12.018	1,947	0.0091

Source: Authors computations using data from CBN, 2021

The heteroskedasticity test, H_0: no ARCH effects vs. H_1: ARCH (p) disturbance are tested. The results from Table 4 shows that the H_0 can be rejected in favour of the four variables considered. DUSD, DPDS, DEUR and DYEN are individually statistically significant at 5% significance level. Hence, the four variables' returns exhibit of an ARCH effect. It is appropriate to apply GARCH model that is sufficient to cope with the changing variance in DUSD, DPDS, DEUR and DYEN. The linear dependence of the series and the order of the model to be fitted are determined by plotting the sample PACF of the square returns for each series.

4.1 *Model Estimation and Diagnostics*

Table 5 displays the outcomes of the joint estimations of the AR, MA, or ARMA model and the GARCH (1, 1) model.

Table 5: Estimation results of fitted GARCH (1,1) Model

	Coeff. Value	Std. Error	Z statistic	P – value
DUSD: AR(1)-GARCH(1,1)			AIC = 4.424	
λ	2.888284	2.73901	1.054499	0.2917
α	-0.00246	0.0000115	-214.833	0.0000
β	0.598058	0.381180	1.568967	0.1167
$\alpha + \beta$ = 1.1711				
DPDS: AR(1)-GARCH(1,1)			AIC = 5.83	
λ	7.68805	0.08619	89.20414	0.0000
α	0.95123	0.024134	39.41406	0.0000
β	-0.003142	0.003699	-0.849595	0.3956
$\alpha + \beta$ = 0.94813				
DEUR: MA(1)-GARCH(1,1)			AIC = 4.50	
λ	2.473293	0.098306	25.15924	0.0000
α	1.681842	0.051880	32.41782	0.0000
β	0.001452	0.004338	0.334793	0.7378
$\alpha + \beta$ = 1.6833				
DYEN: MA(2)-GARCH(1,1)			AIC = -5.337	
λ	0.000120	5.16E-06	23.28724	0.0000
α	1.866944	0.044362	42.08429	0.0000
β	-0.000266	0.003027	-0.087721	0.9301
$\alpha + \beta$ = 1.8667				

The estimates of the mean equation for returns series are all statistically significant and Table 5 displays the variance (volatility) equation result, allowing us to draw the following conclusions:

(i) At 5% level of significance for the DUSD, DPDS, DEUR and DYEN returns series, the first three coefficients, a constant, are statistically significant, while the GARCH term is statistically insignificant, with an expected sign for all returns series. The DUSD series' (constant), which is statistically insignificant at 5% level, is the only notable exception.

(ii) Lagged conditional variance and lagged squared residual have an effect on the conditional variance, as indicated by the significance of both and to put it another way, this indicates that news about volatility (also known as fluctuation) from previous time periods can explain current volatility.

(iii) For each and every time series, the persistence coefficient is calculated as the sum of the two estimated ARCH and GARCH + coefficients. The conditional variance is an explosive process because the persistence coefficient is slightly higher than the return series coefficient. However, in order to have a mean reverting process, the persistence coefficient for the DPDS and DEUR returns series must be less than one. Since the variances are not stationary, these values closer to 1 indicate that volatility shocks are extremely high and will continue.

The AC, PAC, and Q-statistics demonstrate that there is no statistically significant trace of autocorrelation left in the squared standardized residuals, indicating that the mean equation and variance equation are sufficiently specified, making the fitted GARCH model appear to be appropriate for the data presented in Table 6.

Table 6: Tests for the GARCH (1, 1) Models for the returns series

	Squared Standardized Residuals		
	AC	**PAC**	**Q(p-value)**
DUSD	-0.010	-0.0300	49.300 (0.099)
DPDS	0.001	-0.003	3.9311 (1.000)
DEUR	0.036	0.034	11.100 (0.599)
DYEN	-0.008	-0.007	4.1085 (1.000)

As was mentioned earlier, the ARCH and GARCH models do not account for the asymmetry effect, which is the tendency of asset price time series to increase volatility more when there is an unexpected drop than when there is an unexpected increase of the same magnitude. Nelson (1991) proposed the EGARCH model to accommodate asymmetric effects from positive asset returns to negative asset returns.

Table 7: Estimation results of fitted EGARCH (1, 1) Model

	Coeff. Value	Std. Error	Z statistic	P – value
DUSD: AR(1)-EGARCH(1,1) AIC= 4.197				
λ	0.971431	0.043070	22.55490	0.0000
α	-6.709283	0.324198	-20.69503	0.0000
β	6.496661	0.326325	19.90853	0.0000
γ	0.357239	0.029148	12.25594	0.0000
DPDS: AR(1)-EGARCH(1,1) AIC= 5.359				
λ	0.030111	0.037482	0.803338	0.4218
α	0.660241	0.015346	43.02335	0.0000
β	0.032605	0.010386	3.139254	0.0017
γ	0.832481	0.015412	54.01634	0.0000
DEUR: MA(1)- EGARCH(1,1) AIC= 4.478				
λ	0.597054	0.078088	7.645927	0.0000
α	1.242886	0.032169	38.63640	0.0000
β	0.151116	0.020552	7.352904	0.0000
γ	0.163073	0.033528	4.863743	0.0000
DYEN: MA(2)-EGARCH(1,1) AIC = -5.415				
λ	-9.582098	0.119794	-79.98792	0.0000
α	1.306232	0.023595	55.36020	0.0000
β	0.177901	0.013539	13.13971	0.0000
γ	-0.058929	0.014920	-3.949710	0.0001

From Table 7, it is seen that:
(i) At 5% confidence level, all of the coefficients for the DUSD, DPDS, DEUR and DYEN return series are statistically significant. The DPDS return series estimate is the only exception.

(ii) Except for the DYEN returns, where it is negative but significant, the estimates of the leverage effect(s) are positive and significant at 5% confidence level. Therefore, it is evident that volatility is positively correlated with returns in all of our series; that is, lower volatility is accompanied by falling returns, indicating that leverage effects are not present in the daily series during the study period.
In Table 8, the AC, PAC and Q-statistics shows that there is no statistically significant trace of autocorrelation left in the squared standardized residual, indicating that the mean equation and variance equation are adequately specified, making the EGARCH model that has been fitted appear to be appropriate for the data at 1% confidence level.

Table 8: Tests for the EGARCH (1, 1) Models for the Returns Series

	Squared Standardized Residuals		
	AC	**PAC**	**Q (p-value)**
DUSD	-0.007	-0.034	64.470 (0.07)
DPDS	0.003	-0.011	26.921 (0.96)
DEUR	0.042	0.037	18.164 (0.99)
DYEN	-0.012	-0.008	12.769 (1.000)

The fitted TGARCH (1, 1) model to the returns series data is shown in Table 9.

Table 9: Estimation results of fitted TGARCH (1, 1) Model

	Coeff. Value	Std. Error	Z statistic	P – value
DUSD: AR(1)-TGARCH(1,1) AIC= 4.110				
λ	7.48E-05	3.71E-05	2.0159	0.0438
α	0.3519	0.0242	13.9586	0.0000
γ	0.3692	0.0725	5.0939	0.0000
β	0.6731	0.0131	60.4731	0.0000
DPDS: AR(1)-TGARCH(1,1) AIC= 4.701				
λ	4.3931	0.3074	14.9425	0.0000
α	0.0272	0.0494	0.5503	0.5821
γ	2.3947	0.2763	8.6672	0.0000
β	-0.0121	0.0187	-8.6459	0.5183
DEUR: MA(1)-TGARCH(1,1) AIC= 3.911				
λ	2.360994	0.291017	8.112894	0.0000
α	0.009996	0.029625	0.337407	0.7358
γ	0.685040	0.060928	11.24339	0.0000
β	-0.017722	0.080862	-0.219166	0.8265
DYEN: MA(2)-TGARCH(1,1) AIC= -5.521				
λ	0.5230	01451	3.9484	0.0001
α	0.4740	0.1263	3.7539	0.0002
γ	-0.1579	0.1035	-1.5289	0.0273
β	0.5196	0.0843	6.1641	0.0000

From Table 9, it is seen that the estimates of the leverage effect (γ) are positive and significant at 5% confidence level for the returns except for the DYEN returns where it is negative and significant. The positive γ (i.e. good news) indicate that the effect on volatility is σ_1 while the negative γ (i.e. bad news) indicates the effect on volatility is $\alpha_1 + \gamma$. The AC, PAC and Q-statistics show that there is no statistically significant trace of autocorrelation left in the squared standardized residual, indicating that the mean and variance equations are adequately specified, as can be seen from the diagnosis of the goodness of fit of the mean equation and the TGARCH model for the various series data presented in Table 10.

Table 10: Diagnosis for the TGARCH (1, 1) Models for the returns series

Table 10: Tests for the TGARCH (1, 1) Models for the Returns Series

	Squared Standardized Residuals		
	AC	**PAC**	**Q (p-value)**
DUSD	0.011	0.019	17.024 (0.985)
DEUR	0.038	0.035	9.459 (0.977)
DYEN	0.048	0.045	9.469 (0.965)

The fitted IGARCH (1, 1) model to the returns series data is given in Table 11.

Table 11: Estimation results of fitted IGARCH (1, 1) Model

	Coeff. Value	Std. Error	Z statistic	P – value
DUSD: AR(1)-IGARCH(1,1) AIC= 3.991, $\alpha + \beta$ = 1.00315				
α	0.234794	0.005954	39.43515	0.0000
β	0.765206	0.005954	128.5213	0.0000
DPDS: AR(1)-IGARCH(1,1) AIC= 4.621, $\alpha + \beta$ = 0.9999				
α	6.417334	0.004345	29.43515	0.0000
β	-5. 417334	0.005654	28.54213	0.0000
DEUR: MA(1)-IGARCH(1,1) AIC= 3.161, $\alpha + \beta$ = 1.0000				
α	2.522467	0.014345	29.44515	0.0000
β	-1.522467	0.035654	28.44213	0.0000
DYEN: Failure to improve likelihood (non-zero gradients) after 1 iteration.				

The variance (volatility) equation's result is shown in Table 11, and all estimates of the mean equation for returns series are statistically significant. The following can be concluded from Table 11:

(i) The DUSD, DPDS, and DEUR ARCH and GARCH terms have the expected sign for all return series and are statistically significant at 5% level of significance.

(ii) Lagged conditional variance and lagged squared residual have an effect on the conditional variance, as indicated by the significance of both and to put it another way, this indicates that news about volatility (also known as fluctuation) from previous time periods can explain current volatility.

(iii) For each and every time series, the persistence coefficient is calculated as the sum of the two estimated ARCH and GARCH + coefficients. Since the conditional variance is an explosive process, the persistence coefficient is slightly higher than one for the DUSD and DEUR return series. The persistence coefficient in the DPDS return series is lower than one, which is required for a mean-

reverting process. Since the variances are not stationary, these values closer to 1 indicate that volatility shocks are extremely high and will continue.
However, after just one iteration, the study was unable to improve likelihood (non-zero gradients) and therefore was were unable to estimate the IGARCH model for the DYEN return series. The IGARCH model that has been fitted seems appropriate for the data at 1% confidence level because the AC, PAC and Q-statistics show that there is no statistically significant trace of autocorrelation in the squared standardized residual, indicating that the mean and variance equations are adequately specified. This diagnosis of the goodness of fit of the mean equation and the IGARCH model for the various series data is presented in Table 12.

Table 12: Tests for the IGARCH (1, 1) Models for the Returns Series

	Squared Standardized Residuals		
	AC	**PAC**	**Q (p-value)**
DUSD	0.0230	0.0350	59.310 (0.169)
DPDS	0.021	0.0334	7.9211 (1.000)
DEUR	0.076	0.0332	41.110 (1.000)

According to the value of the Akaike Information Criterion (AIC) as summarized in Table 13, the IGARCH modeling technique is preferred for evaluating daily return series for DUSD, DPDS and DEUR, whereas the TGARCH modeling technique is preferred for evaluating daily return series for DYEN.

Table 13: AIC Values for GARCH (1, 1), EGARCH (1, 1), TGARCH (1, 1) and IGARCH (1, 1) Models

	GARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	IGARCH(1,1)
DUSD	4.424	4.197	4.110	3.991
DPDS	5.830	5.359	4.701	4.621
DEUR	4.500	4.478	3.911	3.161
DYEN	-5.337	-5.415	-5.521	-

4.2 *Forecasting using the Selected Models*

Since forecasting is an important application of time series analysis, volatility forecasts for thirty days of the series based on the models chosen by the prescribed information criteria are presented in Figures 13 to 16:
In order to determine the performance of the fitted model in forecasting future volatility, the forecast values are compared with the observed values. From the Table 14, it is evident that the forecast volatility for all the return series converge to the respective sample variance via the observed values of their residual return indicating the ability of the model to predict actual in-sample and out-sample volatility respectively.

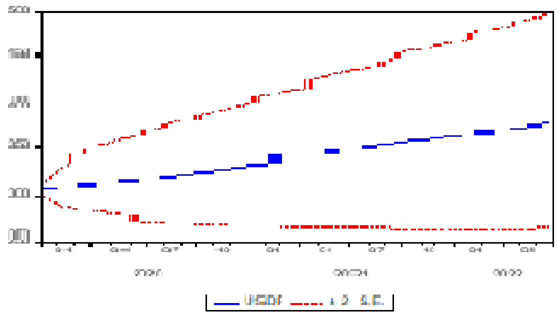

Figure 13: Forecast of USD

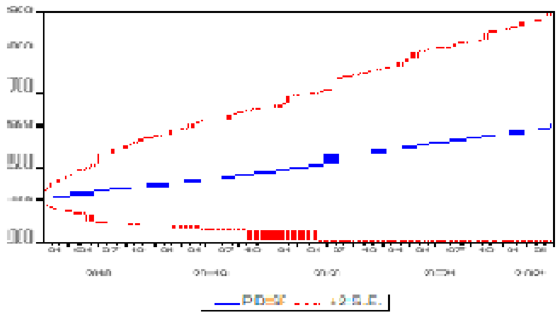

Figure 14: Forecast of PDS

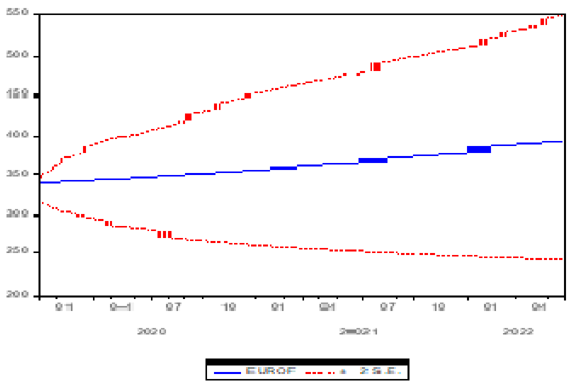

Figure 15: Forecast of EURO

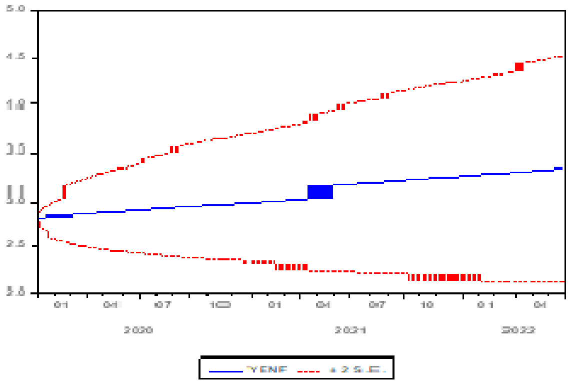

Figure 16: Forecast of YEN

Table 14: Forecast Performance

	Observed values				Forecast values			
Date	USD	PDS	EURO	YEN	USD	PDS	EURO	YEN
1/1/2020	306.95	383.0429	339.7997	2.8442	306.9996115	488.7552	335.0467	2.791975
12/2020	306.95	381.5389	338.4124	2.8546	307.0983757	488.9091	335.1275	2.792735
1/3/2020	306.95	383.2271	337.4608	2.8511	307.1971716	489.0631	335.2083	2.793495
1/6/2020	306.95	380.219	337.7985	2.8567	307.2959993	489.2171	335.2891	2.794255
1/7/2020	306.95	378.9605	337.1232	2.8508	307.3948588	489.3712	335.37	2.795016
1/8/2020	306.95	378.0089	335.6191	2.8382	307.4937501	489.5253	335.4509	2.795777
1/9/2020	307	377.8556	334.64	2.8423	307.5926732	489.6794	335.5318	2.796538
1/10/2020	307	377.3951	335.551	2.8545	307.6916281	489.8336	335.6127	2.797299
1/13/2020	307	378.6538	36.2264	2.8694	307.7906149	489.9879	335.6937	2.79806
1/14/2020	307	378.8073	337.0553	2.8759	307.8896335	490.1422	335.7747	2.798822

5. Conclusion

Exchange rate volatility is an essential metric for a number of financial decision-making models, it has been the focus of numerous empirical studies all over the world. The presence of ARCH/GARCH effects is evident in the plots of the series USD, PDS, EUR and YEN. Also, from residual volatility clustering, clusters contain both large and small errors. This clustering volatility justifies the use of ARCH/GARCH models for series estimation and suggests that the residual term is conditionally heteroscedastic. The presence of the ARCH effect is eventually demonstrated by further testing consequently, the use of the GARCH model. The diagnosis of all GARCH models reveals that there is no autocorrelation in the AC, PAC or Q-statistics, indicating that the mean and variance equations are sufficiently specified. Additional tests of the models were carried out with the series for EGARCH (1, 1), TGARCH (1, 1) and IGARCH (1, 1).

It was discovered that, with the exception of the YEN return series, where the estimate of the leverage effect is negative and significant, volatility is positively correlated with returns in all the series. However, the study was unable to improve likelihood (non-zero gradients) and therefore was unable to estimate the IGARCH model for the DYEN return series after one iteration (see Table 11). It should be noted that one of the worst financial crises in history occurred during the time covered by the data, which may have altered market dynamics.

Declaration

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