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Assessing the Forecast Performance of ARTFIMA-FIAPARCH Hybrid Model

A. G. Umar[1, *] , **H. G. Dikko,** [2] **J. Garba,** [2] **and M. Tasi'u** [2]

[1]*Department of Mathematics, School of Sciences, Kwara State College of Education (T) Lafiagi, Nigeria*
[2]*Department of Statistics, Faculty of Physical Sciences, Ahmadu Bello University, Zaria, Nigeria*

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Abstract. The Long Memory (LM) phenomenon denotes a prolonged association between sequentially gathered observations, characterized by a gradual decline in the autocorrelation function. The Autoregressive Tempered Fractional Integrated Moving Average (ARTFIMA) model addresses non-stationary time series displaying LM in the mean. Conversely, the Fractionally Integrated Asymmetric Power Autoregressive Conditional Heteroscedasticity (FIAPARCH) model is tailored for data exhibiting LM in volatility. This study introduces a novel hybrid model, ARTFIMA-FIAPARCH, employing a transformation method to tackle issues of serial correlation and heteroscedasticity identified in the residuals of the ARTFIMA model. This innovative hybrid model is evaluated using both simulated and real-world data, specifically Naira-Dollar exchange rate data. The assessment involves comparing its performance with existing models like ARFIMA, ARTFIMA and ARFIMA-FIAPARCH based on the minimum Akaike Information Criterion (AIC) and forecast accuracy measures (MAE and RMSE). The findings indicate that ARTFIMA (0,1.3,1.03,3)-FIAPARCH (1,0.08,1) emerges as the superior choice within the ARTFIMA-FIAPARCH models, surpassing ARFIMA (3,1.03,0)-FIAPARCH (1,0.08,1). Conclusively, ARTFIMA-FIAPARCH proves to be a favorable model for examining the mean, volatility and leverage effects of any given economic and financial data. However, it is recommended that economists and financial institutions consider adopting ARTFIMA-FIAPARCH as a viable alternative to existing models.

Keywords: Long Memory, ARFIMA, ARTFIMA, ARTFIMA-FIAPARCH, ARFIMA-FIAPARCH

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*Corresponding author. Email: aminatumar2@gmail.com

1. Introduction

Long Memory behavior refers to a phenomenon where the autocorrelation function of a time series decays very slowly, indicating a strong dependence between observations that are far apart in time. This behavior is distinct from short memory behavior, where the autocorrelation function decays more rapidly. Granger and Joyeux (1980) as well as Hosking (1981) played important roles in examining the statistical properties and implications of long memory. They introduced the concept of fractionally differenced processes, where the differencing parameter can take non integer values. This led to the development of the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, which can capture Long Memory behavior in time series data. The ARFIMA model allows for fractional integration orders, which can better represent the persistence seen in some real-world time series data.

Meerschaert *et al.* (2014) extended the concept further by introducing the Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model. This model accounts for cases where the differencing parameter might be larger than one, which is not typically handled by the ARFIMA model. The tempered parameter in the ARTFIMA model helps control the rate of decay of autocorrelations, allowing for more flexibility in modeling different types of long memory behaviors. Other long memory mean models in literature includes; Semi-parametric Fractional Autoregressive (SEMIFAR) model by Beran (1999), Beta-ARFIMA (β-ARFIMA) model by Pumi *et al.* (2019) and ARFURIMA model by Rahman and Jibrin (2018).

To address long memory in the volatility of a time series data, fractional differencing was also introduced to existing variance models. Nelson (1991) introduced the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model, which was further extended by incorporating fractional differencing. This resulted in the Fractionally Integrated EGARCH (FIEGARCH) model by Bollerslev and Mikkelson (1996). Baillie *et al.* (1996a) introduced fractional differencing into the traditional GARCH model, creating the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) model and Tse (1998) introduced fractional differencing into Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) model of Ding *et al.* (1993) to have FIAPARCH model.

Research has demonstrated that the residuals of non stationary Long Memory mean models, such as ARFIMA and ARTFIMA often exhibit serial correlation. Zhou and He (2009) and Duppati *et al.* (2017) observed this pattern. Similarly, relying solely on a Long Memory variance model like FIAPARCH can lead to unrealistic predictions. To address these limitations, it is important to combine mean and variance models into a hybrid framework which can simultaneously yield improved results. This integration allows for a more comprehensive understanding of the underlying processes governing the data and enhances the accuracy of predictions.

Baillie *et al.* (1996b) made a significant contribution to hybrid modeling by introducing the ARFIMA-GARCH model. They applied this model to study long memory in mean and variance in US inflation concurrently. Ishida and Watanabe (2009) further advanced the research by utilizing conditional sum of squares (CSS) estimators to estimate and compare the ARFIMA-GARCH

model with other hybrid models. These estimators were applied to a large financial time series dataset collected over time. Additionally, Leite *et al.* (2009) and Almeida *et al.* (2017) focused on applying the hybrid ARFIMA-GARCH model to data from the medical field. Their studies revealed that the model performed well in capturing both the long range dependence and volatility present in the medical time series data. Sivakumar and Mohandas (2009) conducted an investigation into the modeling capabilities of the ARFIMA-FIGARCH model using financial data. They compared the results of this hybrid model with those of the ARFIMA model and the comparison revealed that financial market data exhibits both long term memory and volatile characteristics. On a related note, Korkmaz *et al.* (2009) examined the presence of long range dependence in the Istanbul Stock Exchange (ISE) market. They applied the ARFIMA-FIGARCH model to analyze the daily closing prices of ISE transaction prices spanning from 1988 to 2008. The results indicated that there was no apparent evidence of Long Memory in the returns. However, they did observe the presence of four structural breakpoints in the returns. Interestingly, the data did exhibit a Long Memory in terms of volatility.
Recent research has introduced several hybrid models aimed at studying Long Memory in the mean component of time series data. Ambach and Ambach (2018) proposed the ARFIMA-P-GARCH process; Rahman and Jibrin (2018) introduced the Autoregressive Fractional Unit Root Integrated Moving Average-GARCH (ARFURIMA-GARCH) model; and Kabala (2020) introduced the ARTFIMA-GARCH model. Most resent in the study of Long Memory are development of ARFURIMA-APARCH by Jibrin *et al.* (2022) and ARTFIMA-FIGARCH by Umar *et al.* (2023). These models have successfully captured Long Memory in the mean, but they have not explicitly addressed Long Memory in the variance or leverage effect. However, it is important to note that none of these hybrid models have been specifically designed to study Long Memory in the mean, variance and leverage effect concurrently.
In light of this, we introduce a hybrid model called ARTFIMA-FIAPARCH for studying the Long Memory in mean, volatility and leverage effects. The objective is to enhance the accuracy of model fitting and the generation of reliable forecast results.

2. Materials and Method

The general form of an ARFIMA model of Granger and Joyeux (1980) and Hosking (1981) is given by:

$$\varphi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t, \quad 0 < d < 1 \tag{1}$$

The $\varphi(L)$ and $\theta(L)$ are called characteristics polynomial and the $(1-L)^d$ is the fractional differencing filter. The $\varphi_1, \varphi_2, ..., \varphi_p$ and $\theta_1, \theta_2, ...\theta_q$ are unknown parameters and must be estimated from the sample data. d is the Long Memory parameter, L lag operator and ε_t is the error term.
The ARTFIMA model of Meerchaert *et al.* (2014) is defined as follows:

$$\varphi(L)(1-e^{-\lambda}L)^d Y_t = \theta(L)\varepsilon_t \tag{2}$$

where $\varphi(L)$ and $\theta(L)$ are as define in Equation (1), $d > 0$, and $\lambda > 0$ is the tampering parameter. It is also called the stability index for measuring the heavy tail of a time series. The $(1 - e^{-\lambda}L)^d$ is the fractional filter for transforming the nonstationary time series Y_t.
Following Engle (1982), ε_t in Equation (2) is considered to be a stochastic process defined as:

$$\varepsilon_t = a_t \sigma_t \tag{3}$$

where $E(a_t) = 0$, $Var(a_t)$ =1 and σ_t is positive and changes with respect to time, t. This implies that the process $\{a_t\}$, is assumed to be serially uncorrelated and expressed as $a_t \sim iid(0, 1)$

2.1 *Assumptions of the ARTFIMA(p, λ, d, q)-FIAPARCH (1,d,1) Model*

i. The current study assumes that the model in Equation (2) could not completely eliminate the magnitude of trend, heavy tail and Long Memory in the time series $Y_1, Y_2, .., Y_N$ and large proportion of these variations are also found to be present in the residuals $\varepsilon_1, \varepsilon_2$,...,$\varepsilon_N$ of the ARTFIMA model in Equation (2).
ii. Also, the current study assumes that the residuals from the ARTFIMA model, $\varepsilon_1, \varepsilon_2$,...,$\varepsilon_N$ are autocorrelated and heteroscedastic. In time series analysis, estimating the ARTFIMA alone would lead to bad modeling and presenting an unreliable forecast.
iii. The current study assumes that there is substantially high correlation between absolute returns than squared returns, a stylized fact of high frequency financial returns also called Long Memory which is similar to the observations of Safadi and Pereira (2010) and Rahman and Jibrin (2018).

The ARTFIMA-FIAPARCH hybrid model, denoted as ARTFIMA(p, λ, d_1, q)-FIAPARCH$(1, d_2, 1)$, is employed to analyze enduring correlations in mean, volatility, and the leverage effect within time series datasets. Tse (1998) FIAPARCH(1,d,1) model, which forms part of this hybrid, is defined as follows:

$$\sigma_t^2 = \omega[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1}\alpha(L)(1 - L)^d\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \tag{4}$$

Let $\eta^* = \omega[1 - \beta(L)]^{-1}$ and $\tau^* = \{1 - [1 - \beta(L)]^{-1}\alpha(L)(1 - L)^d\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta$

$$\text{Now } \sigma_t = (\eta^* + \tau^*)^{\frac{1}{2}} \tag{5}$$

substituting Equation (5) into Equation (3), we have

$$\varepsilon_t = (\eta^* + \tau^*)^{\frac{1}{2}} a_t \tag{6}$$

Consider ARTFIMA(p, λ, d, q)

$$(1-e^{-\lambda}L)^d Y_t = \sum_{i=1}^{p} \phi_i (1-e^{-\lambda}L)^d Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{7}$$

Also substituting Equation (6) into Equation (7), we have ARTFIMA(p, λ, d_1, q)-FIAPARCH$(1, d_2, 1)$ hybrid model which is represented as:

$$Y_t = \frac{\sum_{i=1}^{p} \phi_i (1-e^{-\lambda}L)^{d_1} Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + (\eta^* + \tau^*)^{\frac{1}{2}} a_t}{(1-e^{-\lambda}L)^{d_1}} \tag{8}$$

where $\phi, \theta, \alpha, \beta, \lambda, \delta, \gamma$ and d_1, d_2 are parameters of the model to be estimated. The $\omega > 0,\ d = d_1 \text{ and } d_2$ are Long Memory parameters for mean and variance models respectively and a_t is the error term for the hybrid model. $\delta \text{ and } \gamma$ are power term and leverage effect parameters respectively.

2.2 *Properties of the Model*

This subsection deals with some properties of the ARTFIMA-FIAPARCH Model.

2.2.1 Mean

Consider ARTFIMA$(1, \lambda, d_1, 1)$-FIAPARCH$(1, d_2, 1)$

$$Y_t = \phi_1 Y_{t-1} + \theta_1 (1-e^{-\lambda}L)^{-d} \varepsilon_{t-1} + (1-e^{-\lambda}L)^{-d} (\eta^* + \tau^*)^{\frac{1}{2}} a_t \tag{9}$$

$$E[Y_t] = \phi_1 E[Y_{t-1}] + \theta_1 (1-e^{-\lambda}L)^{-d} E[\varepsilon_{t-1}] + (1-e^{-\lambda}L)^{-d} (\eta^* + \tau^*)^{\frac{1}{2}} E[a_t] \tag{10}$$

$$\mu = \mu\phi_1 \tag{11}$$

$$\mu - \mu\phi_1 = 0 \tag{12}$$

$$\mu = 0 \tag{13}$$

$E[Y_t] = E[Y_{t-1}] = \mu$ and $E[\varepsilon_{t-1}] = E[a_t] = 0$

$\therefore$ ARTFIMA-FIAPARCH is a zero mean Process.

2.2.2 Variance

To obtain the variance of the model, we first multiply Equation (9) by Y_t and taking its expectation we have:

$$\begin{aligned}&E\left[Y_t^2\right]=\phi_1^2E\left[Y_{t-1}^2\right]+2\phi_1\theta_1(1-e^{-\lambda}L)^{-d}E\left[\varepsilon_{t-1}Y_{t-1}\right]+2\phi_1(1-e^{-\lambda}L)^{-d}\\&\times\sqrt{(\eta^*+\tau^*)}\left[a_tY_{t-1}\right]+2\theta_1((1-e^{-\lambda}L)^{-d}))^2\sqrt{(\eta^*+\tau^*)}E\left[a_t\varepsilon_{t-1}\right]\\&+\theta_1^2((1-e^{-\lambda}L)^{-d}))^2E\left[\varepsilon_{t-1}^2\right]+((1-e^{-\lambda}L)^{-d}))^2(\eta^*+\tau^*)E\left[a_t^2\right]\end{aligned} \tag{14}$$

But $E\left[Y_t^2\right]=E\left[Y_{t-1}^2\right]=\gamma_0$ and $E\left[\varepsilon_{t-1}Y_{t-1}\right] = E\left[a_tY_t\right]=\sigma_\varepsilon^2$

$$\gamma_0=\frac{\sigma_\varepsilon^2\left[2\phi_1\theta_1+\theta_1^2(1-e^{-\lambda}L)^{-d}+(\eta^*+\tau^*)(1-e^{-\lambda}L)^{-d}\right]}{(1-e^{-\lambda}L)^d(1-\phi_1^2)} \tag{15}$$

The variance of ARTFIMA-FIAPARCH Process is given in Equation (15).

2.2.3 Autocovariance at Lag 1

To obtain autocovariance at lag 1, multiply equation (9) Y_{t-1} and taking its expectation we have:

$$E\left[Y_t.Y_{t-1}\right]=E\left[\begin{array}{l}\left[\phi_1Y_{t-1}+\theta_1(1-e^{-\lambda}L)^{-d}\varepsilon_{t-1}+(1-e^{-\lambda}L)^{-d}\sqrt{(\eta^*+\tau^*)}a_t\right]\\\times\left[\phi_1Y_{t-2}+\theta_1(1-e^{-\lambda}L)^{-d}\varepsilon_{t-2}+(1-e^{-\lambda}L)^{-d}\sqrt{(\eta^*+\tau^*)}a_{t-1}\right]\end{array}\right] \tag{16}$$

Simplifying Equation (16) further gives:

$$\gamma_1=\frac{\sigma_\varepsilon^2\left(\phi_1\sqrt{(\eta^*+\tau^*)}+\theta_1(1-e^{-\lambda})^{-d}\right)}{(1-e^{-\lambda}L)^d(1-\phi_1^2)} \tag{17}$$

2.2.4 Autocorrelation at Lag 1 (ρ_1)

$$\rho_1=\frac{\gamma_1}{\gamma_0}$$

$$\rho_1=\frac{\phi_1\sqrt{(\eta^*+\tau^*)}+\theta_1(1-e^{-\lambda}L)^{-d}}{2\phi_1\theta_1+\theta_1^2(1-e^{-\lambda}L)^{-d}+(\eta^*+\tau^*)(1-e^{-\lambda}L)^{-d}} \tag{18}$$

3. Results and Discussion

3.1 *Simulation*

In this section, we generated datasets using Monte Carlo simulation for varying sizes (n = 100, 200, 500, and 1000) for ARTFIMA modeling. Afterwards, we replicated the same datasets for the ARTFIMA-FIAPARCH and ARFIMA-FIAPARCH estimations. For the hybrid models, we performed the estimation process multiple times, exploring different possible combinations of p and q.

Table 1: AIC Values and p-values of Diagnostic Tests for ARTFIMA Model using sample sizes n=100, 200, 500 and 1000

N	ARTFIMA (p, λ,d,q)	AIC	Ljung-Box Test	ARCH-LM Test
	ARTFIMA (1, λ,d,0)	1453.53	0.04199	2.2e-16
	ARTFIMA (0, λ,d,1)	1413.617	0.0003727	2.2e-16
100	ARTFIMA (1, λ,d,1)	1384.246	0.0006916	2.2e-16
	ARTFIMA (2, λ,d,0)	1447.924	0.0005203	2.2e-16
	ARTFIMA (0, λ,d,2)	1413.768	0.0005376	2.2e-16
	ARTFIMA (1, λ,d,0)	735.2701	0.04199	0.0008
	ARTFIMA (0, λ,d,1)	735.3077	0.0003727	0.0005
200	ARTFIMA (1, λ,d,1)	737.2214	0.0006916	0.0001
	ARTFIMA (2, λ,d,0)	738.0678	0.0005203	0.0001
	ARTFIMA (0, λ,d,2)	737.784	0.0005376	2.2e-16
	ARTFIMA (1, λ,d,0)	7284.098	0.02875	0.0044
	ARTFIMA (0, λ,d,1)	7085.71	3.163e-07	0.00023
500	ARTFIMA (1, λ,d,1)	7087.665	3.075e-07	0.0249
	ARTFIMA (2, λ,d,0)	7237.371	0.003882	0.0004
	ARTFIMA (0, λ,d,2)	7090.789	4.161e-07	2.2e-16
	ARTFIMA (1, λ,d,0)	14567.71	0.0002619	2.2e-16
	ARTFIMA (0, λ,d,1)	14172.75	1.854e-12	2.2e-16
1000	ARTFIMA (1, λ,d,1)	14174.68	1.953e-12	2.2e-16
	ARTFIMA (2, λ,d,0)	14449.01	5.589e-06	2.2e-16
	ARTFIMA (0, λ,d,2)	14181.66	2.064e-12	2.2e-16

The results obtained from performing the Ljung-Box Test and ARCH-LM Test on the simulated data sets, as presented in Table 1 show p-values less than 0.05 which indicate that the residuals of the ARTFIMA model show evidence of serial correlation and heteroscedasticity. This suggests that the ARTFIMA model alone may not be sufficient to adequately capture the complexities in the simulated datasets and there is a need to enhance it by combining it with a variance model. To address this issue, FIAPARCH variance model is considered and integrated into the ARTFIMA model. The aim for incorporating this variance model is to minimize errors and remove the serial correlation and heteroscedasticity in the data which ultimately will lead to an improved and more accurate representation of the underlying process.

Table 2: AIC Values and p-values for Diagnostic Tests for ARFIMA-FIAPARCH and ARTFIMA-FIAPARCH Model using sample sizes n=100, 200, 500 and 1000

N	Model	AIC	Ljung-Box	ARCH-LM	Model	AIC	Ljung-Box	ARCHLM
	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	**14.328**	**0.802**	**0.698**	ARTFIMA(1, λ,d_1,0)-FIAPARCH(1,d_2,1)	-	-	-
	ARFIMA(0,d_1,1)-FIAPARCH(1,d_2,1)	14.328	0.832	0.679	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.326**	**0.877**	**0.6734**
100	ARFIMA(1 ,d_1,1)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(1, λ,d_1,1)-FIAPARCH(1,d_2,1)	14.352	0.977	0.4013
	ARFIMA(2,d_1,0)-FIAPARCH(1,d_2,1)	14.348	0.273	0.701	ARTFIMA(2, λ,d_1,0)-FIAPARCH(1,d_2,1)	14.348	0.346	0.9422
	ARFIMA(0,d_1,2)-FIAPARCH(1,d_2,1)	14.347	0.288	0.683	ARTFIMA(0, λ,d_1,2)-FIAPARCH(1,d_2,1)	14.347	-	-
	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	14.211	0.891	0.845	ARTFIMA(1, λ,d_1,0)-FIAPARCH(1,d_2,1)	14.211	0.044	0.1256
	ARFIMA(0,d_1,1)-FIAPARCH(1,d_2,1)	14.210	0.951	0.849	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.152**	**0.765**	**0.9765**
200	**ARFIMA(1,d_1,1)-FIAPARCH(1,d_2,1)**	**14.195**	**0.689**	**0.720**	ARTFIMA(1, λ,d1,1)-FIAPARCH(1,d_2,1)	-	-	-
	ARFIMA(2,d_1,0)-FIAPARCH(1,d_2,1)	14.218	0.794	0.915	ARTFIMA(2, λ,d_1,0)-FIAPARCH(1,d_2,1)	-	-	-
	ARFIMA(0,d_1,2)-FIAPARCH(1,d_2,1)	14.217	0.937	0.916	ARTFIMA(0, λ,d_1,2)-FIAPARCH(1,d_2,1)	14.188	0.8765	0.7654
	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	**14.212**	**0.979**	**0.887**	ARTFIMA(1, λ,d_1,0)-FIAPARCH(1,d_2,1)	- -		
	ARFIMA(0,d_1,1)-FIAPARCH(1,d_2,1)	14.212	0.979	0.888	ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)	14.249	0.672	0.8041
500	ARFIMA(1,d_1,1)-FIAPARCH(1,d_2,1)	14.216	0.979	0.889	**ARTFIMA(1, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.201**	-	-
	ARFIMA(2,d_1,0)-FIAPARCH(1,d_2,1)	14.216	0.979	0.897	ARTFIMA(2λ,d_1,0)-FIAPARCH(1,d_2,1)	-	-	-
	ARFIMA(0,d_1,2)-FIAPARCH(1,d2,1)	14.216	0.979	0.893	ARTFIMA(0, λ,d_1,2)-FIAPARCH(1,d_2,1)	14.253	0.999	0.6837
	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	14.198	0.952	0.167	ARTFIMA(1, λ,d_1,0)-FIAPARCH(1,d_2,1)	14.579	0.898	0.2448
	ARFIMA(0,d_1,1)-FIAPARCH(1,d_2,1)	14.237	0.761	0.217	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.181**	**0.763**	**0.4536**
1000	ARFIMA(1,d_1,1)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(1, λ,d_1,1)-FIAPARCH(1,d_2,1)	-	-	-
	ARFIMA(2,d_1,0)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(2, λ,d_1,0)-FIAPARCH(1,d_2,1)	14.459	0.95	0.5164
	ARFIMA(0,d_1,2)-FIAPARCH(1,d_2,1)	**14.183**	**0.951**	**0.156**	ARTFIMA(0, λ,d_1,2)-FIAPARCH(1,d_2,1)	-		

In summary, the diagnostic assessment across the simulated datasets in Table 2 indicates lower AIC values in comparison to the ARTFIMA mean model in Table 1. The p-values associated to the two hybrid models are less than 0.05 therefore exhibited no discernible patterns and the variability of the residuals seemed consistent across the datasets.

Table 3: Estimation of ARFIMA(p,d1,q) FIAPARCH(1,d2,1) and ARTFIMA(p,λ,d_1,q)-FIAPARCH(1,d_2,1) with AIC Values and Measures of Forecast Accuracy Values using Simulated Datasets

N	Model	AIC	MAE	RMSE	Model	AIC	MAE	RMSE
100	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	14.328	207.275	250.428	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.326**	**0.856**	**1.055**
200	ARFIMA(1,d_1,1)-FIAPARCH(1,d_2,1)	14.195	234.253	270.223	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.152**	**0.845**	**1.329**
500	ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	14.212	248.607	287.407	**ARTFIMA(1, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.201**	**0.852**	**0.987**
1000	ARFIMA(0,d_1,2)-FIAPARCH(1,d_2,1)	14.183	250.004	288.512	**ARTFIMA(0, λ,d_1,1)-FIAPARCH(1,d_2,1)**	**14.181**	**0.867**	**1.002**

In summary, the results from Table 3 indicate that the ARTFIMA-FIAPARCH model outperforms the existing ARFIMA-FIAPARCH model in terms of forecasting accuracy. Therefore, the incorporation of the FIAPARCH variance model into the ARTFIMA model has led to significant improvements, making the hybrid model a more suitable choice for analyzing the simulated data and making more accurate predictions.

3.2 *Application*

This section presents the application of the ARTFIMA, ARFIMA, ARTFIMA-FIAPARCH and ARFIMA-FIAPARCH model by using daily exchange rate data of Nigeria Naira (NGN) Vs US Dollar (USD) obtained from cbn.gov.ng from 2nd January 2013 to 23rd May 2023.

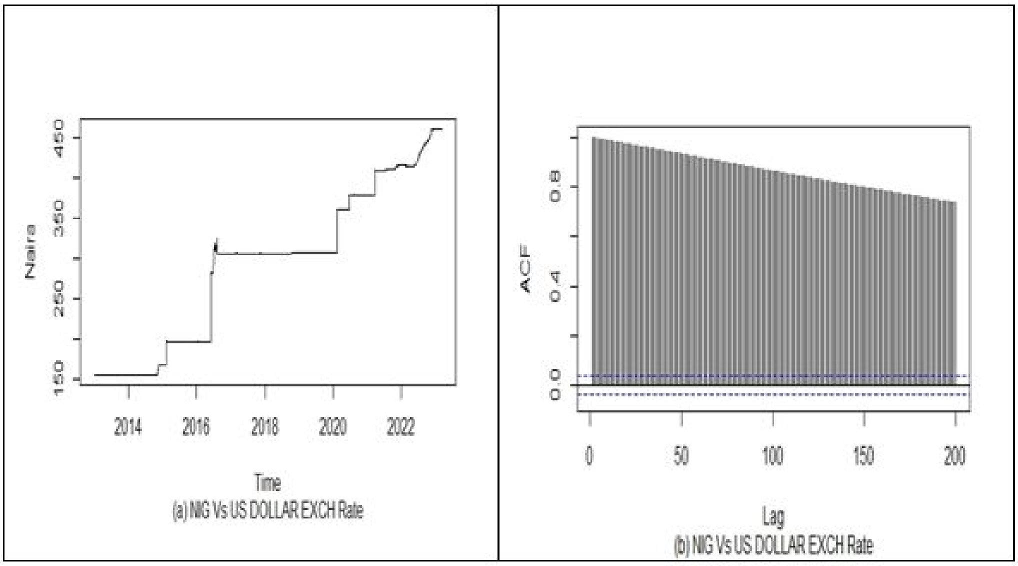

Figure 1: Time Series Plot and ACF for daily NGN Vs USD Exchange Rate.

Figure 1 Illustrates a continuous decrease in the chart of the NGN Vs USD exchange rate from 2013 to 2023 highlights a persistent devaluation of the Naira against the Dollar. Despite occasional fluctuations, the overall trajectory signifies a consistent weakening of the naira. Multiple factors contribute to this pattern, including the decline in oil prices, a substantial current account deficit, and governmental monetary policies. The repercussions of the naira's devaluation have adversely impacted the Nigerian economy, leading to various negative effects. The autocorrelation function (ACF) plot for the NGN Vs USD exchange rate illustrates a prominent positive correlation, signaling a substantial and strong association between the current exchange rate and the rate observed on the preceding day. The ACF plot implies that the time series of the NGN Vs USD exchange rate does not exhibit stationarity, indicating a non-stationary characteristic and the existence of certain Long Memory effects.

3.3 *Mean Modeling*

This section focuses on identifying the parameters for each set of candidate mean models, namely the ARFIMA and ARTFIMA models. The goal is to determine the appropriate parameter values that best capture the characteristics of the data.

ARFIMA and ARTFIMA Mean Models Identification

Table 4 presents the results of the analysis conducted on the NGN vs. USD exchange rate dataset using two mean models: ARFIMA and ARTFIMA. The table shows the estimated parameter values for each model, which represent the specific values chosen to characterize the mean component of the time series data. These parameter estimates are crucial in understanding and describing how the mean of the exchange rate series behaves over time for both models.

Table 4: AIC and Diagnostic tests P-values for ARFIMA(p,d,q) and ARTFIMA (p,λ,d,q) Models

ARFIMA(p,d,q)				ARTFIMA (p,λ,d,q)			
ARFIMA (p,1.3,q)	AIC	Ljung-Box Test	ARCH-LM TestB	ARTFIMA (p,1.3,1.03,q)	AIC	Ljung-Box Test	ARCH-LM Test
ARFIMA (1,d,0)	**11432.9**	0.0003	2.2e-16	ARTFIMA (1, λ,d,0)	**11379.8**	0.0004	2.2e-16
ARFIMA (0,d,1)	14779.5	0.0003	2.2e-16	ARTFIMA (0, λ,d,1)	11379.8	0.0004	2.2e-16
ARFIMA (1,d,1)	11434.9	0.0004	2.2e-16	ARTFIMA (1, λ,d,1)	11380.8	0.0007	2.2e-16
ARFIMA (2,d,0)	11435.8	0.0003	2.2e-16	ARTFIMA (2, λ,d,0)	11382.0	0.0005	2.2e-16
ARFIMA (0,d,2)	13793.6	3.195e-05	2.2e-16	ARTFIMA (0, λ,d,2)	11381.9	0.0005	2.2e-16

The AIC values are compared to determine the best-fit models, and it was found that the ARTFIMA models had the lowest AIC values. This suggests that the ARTFIMA models are more suitable for the data compared to the ARFIMA models. However, upon examining the residuals of both the ARFIMA and ARTFIMA models, it is observed that they exhibited evidence of serial correlation and heteroscedasticity. This is confirmed by having p-values that are less than 0.05, indicating significant departures from normality. To address these issues and improve model fitting, the study considered incorporating fractionally integrated volatility models, specifically the FIAPARCH models, into both the ARFIMA and ARTFIMA models. The inclusion of FIAPARCH variance model aims to better capture the volatility characteristics of the data. Further analyses are conducted based on the hybrid models of ARFIMA-FIAPARCH and ARTFIMA-FIAPARCH to improve the overall models' fit to the data.

3.4 *The Hybrid Models Identification*

The estimation procedures for the two hybrid models are repeated for multiple iterations i.e $p \leq 3$ and $q \leq= 3$ while it is assumed that the residuals of models follow a normal (norm) distribution. Akaike Information Criteria (AIC) is used to select the optimal models and the performance of the models are evaluated using forecast accuracy measures such as Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) for basis of comparison. Following the diagnostic tests and model identification, the most optimal models for the two hybrid models are identified for daily NGN Vs USD exchange rate data.

Table 5: The Estimation of ARFIMA(p,1.03,q) FIAPARCH(1,0.08,1) and ARTFIMA(p,1.5,1.03,q)-FIAPARCH(1,0.08,1) with their AIC and Diagnostic Test P-values using NGN Vs USD Data

ARFIMA(p,1.03,q)-FIAPARCH(1,0.08,1)				ARTFIMA(p,1.5,1.03,q)-FIAPARCH(1,0.08,1)			
ARFIMA(1,d_1,0)-FIAPARCH(1,d_2,1)	-1.828	0.946	0.971	ARTFIMA(1, λ, d_1,0)-FIAPARCH(1,d_2,1)	-1.834	0.965	0.961
ARFIMA(0, d_1,1)-FIAPARCH(1,d_2,1)	7.182	0.889	0.984	ARTFIMA(0, λ, d_1,1)-FIAPARCH(1,d_2,1)	-1.832	0.973	0.963
ARFIMA(1, d_1,1)-FIAPARCH(1,d_2,1)	-2.701	0.985	0.969	ARTFIMA(1, λ, d_1,1)-FIAPARCH(1,d_2,1)	-1.808	0.921	0.965
ARFIMA(2, d_1,0)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(2, λ, d_1,0)-FIAPARCH(1,d_2,1)	-3.193	0.961	0.973
ARFIMA(0, d_1,2)-FIAPARCH(1,d_2,1)	6.329	0.919	0.984	ARTFIMA(0, λ, d_1,2)-FIAPARCH(1,d_2,1)	-1.864	0.981	0.963
ARFIMA(2, d_1,1)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(2, λ, d_1,1)-FIAPARCH(1,d_2,1)	-2.809	0.954	0.973
ARFIMA(1, d_1,2)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(1, λ, d_1,2)-FIAPARCH(1,d_2,1)	-1.872	0.8811	0.9576
ARFIMA(3, d_1,0)-FIAPARCH(1,d_2,1)	**-2.771**	**0.987**	**0.969**	ARTFIMA(3, λ, d_1,0)-FIAPARCH(1,d_2,1)	-2.998	0.962	0.973
ARFIMA(0, d1,3)-FIAPARCH(1,d2,1)	5.676	0.974	0.984	**ARTFIMA(0, λ, d_1,3)-FIAPARCH(1,d_2,1)**	**-3.281**	**0.957**	**0.973**
ARFIMA(3, d_1,1)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(3, λ, d_1,1)-FIAPARCH(1,d_2,1)	-3.153	0.943	0.973
ARFIMA(1, d_1,3)-FIAPARCH(1,d_2,1)	-2.548	0.983	0.983	ARTFIMA(1, λ, d_1,3)-FIAPARCH(1,d_2,1)	-1.873	0.988	0.961
ARFIMA(2, d_1,2)-FIAPARCH(1,d_2,1)	-1.719	0.207	0.975	ARTFIMA(2, λ, d_1,2)-FIAPARCH(1,d_2,1)	-1.896	0.979	0.958
ARFIMA(3, d_1,2)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(3, λ, d_1,2)-FIAPARCH(1,d_2,1)	-1.889	0.976	0.963
ARFIMA(2, d_1,3)-FIAPARCH(1,d_2,1)	-	-	-	ARTFIMA(2, λ, d_1,3)-FIAPARCH(1,d_2,1)	-1.893	0.982	0.957

Table 5 showcases the results obtained by applying the ARFIMA-FIAPARCH and ARTFIMA-FIAPARCH hybrid models to the NGN vs. USD dataset. Notably, both models exhibit residuals that lack discernible patterns or systematic relationships. Furthermore, the variability in the residuals remains constant across the entire dataset, indicating homoscedasticity. Additionally, the absence of serial correlation (diagnostic tests p-values ¡ 0.05) in the residuals reinforces the reliability of both the ARFIMA-FIAPARCH and ARTFIMA-FIAPARCH models. The homoscedastic and nonserially correlated nature of the residuals in both models contributes to their robustness and reinforces their potential utility in capturing and explaining the observed patterns in the NGN vs. USD exchange rate.

Table 6: Estimation of ARFIMA(p,d,q) –FIAPARCH(1,1) and ARTFIMA (p,λ,d,q)-FIAPARCH(1,1) with AIC Values

Model	Parameters	Estimate	P-value	AIC	MAE	RMSE
ARTFIMA (0,1.3,1.03,3)-FIAPARCH (1,0.08,1)	θ_1	-0.208	0.000	-12.27	0.299	3.589
	θ_2	-0.272	0.000			
	θ_3	0.102	0.000			
	α	0.286	0.000			
	β	0.742	0.000			
	γ	0.444	0.000			
	δ	-0.152	0.000			
ARFIMA (3,1.03,0)-FIAPARCH (1,0.08,1)	ϕ_1	0.763	0.000	-5.921	6.745	6.226
	ϕ_2	0.597	0.000			
	ϕ_3	-0.360	0.000			
	α	0.314	0.000			
	β	0.715	0.000			
	γ	0.406	0.000			
	δ	-0.326	0.000			

Table 6 Provides forecast accuracy and parameter estimates for NGN versus USD using hybrid models ARTFIMA-FIAPARCH and ARFIMA-FIAPARCH. Within these models, ARTFIMA (0, 1.3, 1.03, 3)-FIAPARCH (1, 0.08, 1) display lower forecast accuracy values compared to its counterpart. Notably, only

the moving average part of the mean model is statistically significant. The estimated moving average coefficients (θ_1, θ_2, and θ_3) indicate the impact of error terms at lags 1, 2, and 3. Regarding volatility, the model's parameters offer meaningful insights. The values α = 0.286 and β = 0.742 signify a substantial influence of recent squared errors on current volatility, underscoring the strong impact of past squared errors on present volatility. while γ = 0.444 indicates a moderate positive impact of long-term past volatility on the current volatility. This implies that past periods of high volatility have a notable effect on the current volatility and finally, δ= -0.152 reflects an asymmetric response, with negative shocks having a larger impact on volatility than positive shocks of NGN vs USD exchange rate.

4. Conclusion

The simulation study conducted yielded valuable insights into the performance of the newly developed model, and these insights are further validated by the results obtained from a real-life dataset, as outlined in Table 6. Upon examination of the result, it is evident that ARTFIMA (0,1.3,1.03,3)-FIAPARCH (1,0.03,1) demonstrates significantly lower forecast accuracy values compared to its counterpart, ARFIMA (3,1.03,0)-FIAPARCH (1,0.03,1). This consistent pattern observed across various simulated datasets and the real-life data underscores the performance of ARTFIMA-FIAPARCH hybrid model in minimizing errors when compared to ARFIMA-FIAPARCH. In summary, the comprehensive assessment of results across all datasets consistently supports the conclusion that the ARTFIMA-FIAPARCH hybrid model exhibits superior forecasting accuracy by minimizing errors in comparison to its established counterpart, ARFIMA-FIAPARCH. This validation establishes the new model as a better fit for the examined real-life dataset. The practical implications of this finding are substantial, suggesting that the ARTFIMA-FIAPARCH model can offer more reliable and accurate predictions, making it a valuable tool for forecasting NGN Vs USD exchange rate characterized by long memory in mean, volatility, and leverage effects.

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