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A Study on Non-Homogenous Markov Fuzzy Manpower Systems

V. O. Ezugwu[1, *], **A. A. Osagiede** [2], **and V. A. Amenaghawon** [3]

[1]*Department of Statistics, University of Uyo, Uyo, Nigeria*
[2]*Department of Mathematics, University of Benin, Nigeria*
[3]*Department of Computer Science and Information Teechnology, Igbinedion University, Okada, Nigeria*

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Abstract. There are available research works which considered intra-state heterogeneity in personnel transition behavior due to latent factors in literature. None of these research works has captured any specific latent factor or combination of specific latent factors which influence the within-state transition differences in a manpower model. This work considers a hierarchical non-homogeneous manpower system in which promotion of employees is only assessable based on the level of innovativeness and job performance capability. For this system, a non-homogeneous Markov model which takes into account theory of fuzzy sets is proposed. The model is proposed to deal with the problem of vagueness associated with gradual transition of members between the crisp states of the manpower system. It is also, proposed to incorporate key personality traits which influence employees within a homogeneous category to behave in different ways. The total transition probability matrix is estimated. The limiting probability structure for the fuzzy manpower system is obtained as [0.2181,0.2233,0.20280,0.3505]. This suggests that greater proportion of staff would possess advance level of openness and advance level of conscientiousness in the long run compared to other levels of combinations of personnel traits.

Keywords: Fuzzy Set, Fuzzy state, Fuzzy Manpower System, Membership Function, Non-Homogeneous Markov Chain.

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1. Introduction

Human Resources Planning (HRP) represents the range of philosophies, tools and techniques that any organization should deploy to monitor and manage the movement of staff, both in terms of numbers and profiles, (Behlaji and Tkiouat 2013). Manpower planning is concerned with personnel supply-and-demand

*Corresponding author. Email: vitusezugwu@uniuyo.edu.ng

prediction and development of personnel strategy which ensures that the required personnel is available at the right time, (Bartholomew et at, 1991).

Fuzzy set is a set that does not have clearly defined boundaries (limits) and can contain members only at some degree. Fuzzy set theory is an extension of classical set theory proposed by (Zadeh, 1968), that provides a mathematical framework for handling categories that permit partial membership (or membership in degree). The fuzzy state in a Non-Homogeneous Markov System (NHMS) is defined mathematically by assigning to each possible member of the state a value representing its grade of membership in the fuzzy state (Symeonaki et al, 2002). Thus, fuzzy manpower system is a manpower system that consists of fuzzy states. In aggregate, the workforce system of any organisation comprises of a stock of heterogeneous personnel. In most manpower models, such as; (Belhaj and Tkiouat 2013; Ezugwu and Igbinosun, 2020; Vassiliou, 2021), the manpower system is hierarchically graded into mutually exclusive and exhaustive grades so that each member of the system may belong to one and only one of the grades at any given time. The aggregated personnel system is partitioned into homogeneous groups in such a way that members of staff in the same grade have certain common attributes, (De Feyter 2006), and are presumed to evolve analogously.

A popular mathematical model for modelling manpower systems is Markovian model. Markov chain theory is widely used in manpower planning for forecasting, as well as control of personnel structure, (Symeonaki and Stamoua, 2004; Jeeva and Geetha, 2013). Markov chain theory is also used in portfolio allocation and market equilibrium mix; (Ezugwu et al, 2013; Ezugwu and Igbinosun 2016). For these manpower models, the aggregated personnel system is classified into homogeneous groups on the basis of whatever attributes that are relevant for the problem at hand. Concerning Markovian approach for modelling manpower systems, Markov models are classified into homogeneous or non-homogeneous, based on the nature of the system's dependency on time. A Markov chain model is said to be homogeneous if the transition probabilities of the members are assumed to be independent of time. Examples include; (Ekhosuehi and Osagiede, 2006; Ekhosuehi et al. 2017; Ezugwu and Ologun 2017; Ezugwu and Igbinosun 2020). A Markov chain model is said to be non-homogeneous if the transition probabilities of the members are assumed to be dependent on time. Examples include (Vassiliou, 2021: Assi and Effanga, 2022; Vassiliou, 2022).

In this study, the concept of non-homogeneous Markov manpower system is considered. The concept of Non-homogeneous Markov systems was first introduced in (Vassiliou, 1982) and the motive was to provide a more general framework for a number of Non-homogeneous Markov chain models in manpower systems (Vassiliou, 2018). In this study, we examine the necessity of introducing fuzzy states in non-homogeneous Markov manpower systems. Like it was earlier stressed, manpower planning analysis based on Markovian approach assumes that the system of interest is partitioned into distinct classes (states), where each member of the system "clearly" belongs to one and only one of the classes at time t, and makes transition from one states to another at time $t + 1$. In other word, Markovian approach requires that the states of the system under study be precisely measured and defined in such a way that the members

of the system are dichotomized into two groups: members and non-members. However, this assumption is unrealistic in some situations regarding manpower system's classification. In some situations in real application of Markov theory in manpower planning analysis, one is often faced with the fact of fuzzy states, in the sense that the states of the system cannot be precisely measured due to vagueness in transition of members from particular state to another. This method of classification of states of Markov system into fuzzy states can also be appropriate for manpower systems. For instance, different personnel belonging to the same grade do exhibit different rates of transition to the next higher grades of a hierarchical manpower system.
In hierarchical manpower system, promotion of a member of the system from current grade to the next higher grade is normally possible after the member has completed all the necessary requirements for promotion, peculiar to that particular origin state. It is realistic that at a particular time t, different members of the same grade have different current levels of completion of promotion requirements peculiar to the state, which guarantee their next promotion to the next higher grade at different times $t+1, t+2, \ldots$. It may, however be unrealistic to assume or project a uniform transition period for every member of a particular grade. This indicates ambiguity concerning membership of the same state of the system, and should better be perceived as having imprecise boundaries that facilitate gradual transition from membership to non-membership, and vice versa.
In the previous works, a method to deal with problem of lack of observations for some variable: by building of a hidden Markov model or Markov switching model was introduced, (Udom and Ebedoro, 2019). Also introduced is a method that takes into account latent sources of heterogeneity in manpower systems, (Ugwuowo and McMclean, 2000). However, concerning hidden Markov model for manpower planning analysis, observation shows that transitions from the latent states are not free of ambiguity. For instance, in (Guerry,2011; Udom and Ebedoro, 2019), there is no clear cut concerning the value of transition probabilities for members of (movers and stayers) latent subclasses common to the entire personnel categories. Thus, the definition of the latent subclasses is not precise. It is subject to vagueness due to the fact that the value of probability which qualifies an individual to belong to each of the distinguishable latent subclasses is not common to the entire personnel categories. Therefore, real applications of Markov Models in manpower planning indicates strongly the need for introducing a new method of estimating the above mentioned probabilities, which is the prime motivating factor for considering fuzzy logic and fuzzy reasoning in non-homogeneous Markov manpower systems.
Steady State Conditions in Tractable Markov Manpower Model for an Extended Manpower System was discussed by (Ossai, 2023). In his work, he formulated a manpower structure in discrete time homogeneous Markov model for a multi level manpower system. (Vassiliou, 2024) studied the problem of strong ergodicity in non-homogeneous Markov system. In his work, he relaxed the fundamental assumption present in all studies of asymptotic behavior. That is, the assumption that the inherent inhomgeneous Markov chain converges to a homogeneous Markov chain with regular transition probability matrix. (Vassiliou, 2022), studied Limiting Distributions of the Non-Homogeneous Markov

System in Stochastic Environment in Continuous Time. In the paper, he stated that ordinary non-homogeneous Markov process is a very special case of an Non-Homogeneous Markov Systems in a Stochastic Environment in Continuous Time (S-NHMSC). He then studied the expected population structure of S-NHMSC. The first central classical problem was, finding the condition under which the asymptotic behavior of the expected population structure exists. The second central problem was, finding which expected population structures are possible limiting ones provided the limiting vector of input probabilities into the population is controlled. (Agboola and Ahmad, 2023) considered a Markovian approach in studying the behavior of academic staff grade levels transitions in private university in Nigeria. The purpose was to determine the proportion of staff recruited, promoted and withdrawn from various grade levels in the private university from 2022/2023 to 2030/2031 academic sessions. However, in the real world, manpower system possesses a number of imprecise and dynamic humanistic factors which play a significant role in their overall behaviors.

Consequently, most of the decision making takes place in a dynamic fuzzy environment in which the goals, the constraints and the impacts of possible actions are not precisely known. The concept of a fuzzy non-homogeneous Markov system (F-NHMS) was introduced and defined for the first time in (Symeonaki et al, 2002). In the study, in an effort to deal with the uncertainty introduced in the estimation of transition probabilities and the input probabilities in Markov systems, the theory of fuzzy logic and fuzzy reasoning was combined with the theory of Markov system and the concept of a fuzzy non-homogeneous Markov system was introduced. A handful of papers (Guerry, 2011) have devoted to partitioning personnel systems based on these latent factor, to handle their sources of personnel differences. (Guerry, 2011) discussed hidden heterogeneity in manpower Systems: a Markov-switching model approach. In this work, a two-step procedure was introduced for incorporating personnel heterogeneity into manpower modeling. Thus, for this present study, a fuzzy set theory is introduced to incorporate specific latent factors (individual traits) in the analysis of manpower systems based on the concept of non-homogeneous Markov theory. Fuzzy partitioning is introduced to classify individuals in each personnel category (determined by observable variables) into fuzzy states on the basis of (Advance and Naive) levels of combination of a pair of specific individual traits (latent attributes).

We first consider a manpower system which is stratified into categories (states) based on the organizational attribute of interest, say grade. Let $G_1, G_2, \ldots, G_k$ be the set of states that are assumed to be exhaustive and exclusive. Consider a discrete time scale $t = 1, 2, \ldots$, and denote the structure of the system at any given time t by the row vector, $N(t) = [N_1(t), N_2(t), \ldots, N_k(t)]$, where $N_i(t)$ is the expected number of members in grade $G_i (i = 1, 2, \ldots, k)$ at time t. Denote also $\{T(t)\}_{t=1}^{\infty}$ to be a sequence indicating the total number of members in the system at time t, and $\Delta T(t) = T(t + 1) - T(t)$. Let $\{P(t)\}_{t=1}^{\infty}$ be the sequence of transition probability matrices between states,$\{P_0(t)\}_{t=1}^{\infty}$, the sequence of vectors of probabilities of allocating new recruits to the state, G_i, and $\{P_{k+1}(t)\}_{t=1}^{\infty}$, the sequence of vectors of probabilities of wastages from the grades G_i of the system. Let $Q(t) = P(t) + P'_{k+1}(t)P_0(t)$, where $(\cdot)'$ denotes the transpose of the recruitment vector, then $Q(t)$ is a stochastic matrix known as the total transition probability and $\{Q(t)\}_{t=1}^{\infty}$ defines what is called an embedded non-homogeneous Markov chain. The expected number of members in the various states at time t can be obtained from the following equation: $N(t) = N(t-1)P(t-1) + \Delta t(t-1)P_0(t-1)$. The system described above is called a Non-Homogeneous Markov manpower system (Vassiliou, 2018).

2. Materials and Method

Let the aggregated manpower system of the organization be partitioned into categories based on a certain attribute of interest, say grade. Let $G_i (i = 1, 2, \ldots k)$ denote the crisp states (or the grades of the system) that are assumed to be mutually exclusive and collectively exhaustive, where k is the highest of the hierarchical grades. Let G_0 a wastage category, represent external environment to which any member who leaves the system is transferred. In the analysis of differentials in manpower systems, (Ugwuowo and McClean, 2000), sources of personnel differences were classified into observable and Latent sources. The latent sources were classified into individual traits and environmental factors (Ugwuowo and McClean. 2000). However, for the purpose of this study, we restrict environmental factors only to organizational culture, and assume that the influence of organizational culture on individual career development is homogeneous for every member of the system. In any organizational manpower system, individual traits are very diverse, and as such, the influence of individual traits on career development (or progress) is also very diverse for various members of the organization. In personality study, individual traits are partitioned (Ali, 2017) into five classes, viz; Openness, Conscientiousness, Extraversion, Agreeableness, and Neuroticism. For the purpose of this work, Openness and Conscientiousness are considered. It is assumed that individual transitions behavior between the personnel categories is a function of the levels (naive and advance) of combination of these personnel traits. Thus, the fuzzy states for this work are partitioned based on these combinations. Fuzzy partitions are linguistic representations of their universe of discourse, (Symeonaki and Stamou, 2004). Their elements are linguistic terms like low, medium, high, etc. For this work, the fuzzy partitions are formulated in terms of Naive and Advance levels of the combination of the aforementioned individual traits. Therefore we consider that $F = F_1, F_2, F_3, F_4$ is the fuzzy state space of the system, where F_1 describes the combination of naive level of openness and naive level of conscientiousness, F_2 describes advance level of openness and naive level of conscientiousness, F_3 describes naive level of openness and advance level of conscientiousness, while F_4 describes advance level of openness and advance level of conscientiousness. Let Π be a $k \times 4$ matrix of the membership values for the fuzzy states, then

$$\Pi = \begin{bmatrix} \mu_{F_1}(1) & \mu_{F_2}(1) & \mu_{F_3}(1) & \mu_{F_4}(1) \\ \mu_{F_1}(2) & \mu_{F_2}(2) & \mu_{F_3}(2) & \mu_{F_4}(2) \\ \mu_{F_1}(3) & \mu_{F_2}(3) & \mu_{F_3}(3) & \mu_{F_4}(3) \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{F_1}(k) & \mu_{F_2}(k) & \mu_{F_3}(k) & \mu_{F_4}(k) \end{bmatrix} \tag{1}$$

Definition 1: Given two fuzzy events, A and B, with $\mu_{F_A}(\cdot)$ and $\mu_{F_B}(\cdot)$ being the membership functions of event A and event B, respectively. The product of the two fuzzy events (sets) A and B is defined by (Bhattacharyya 1998) as

$$A \cdot B \longleftrightarrow \mu_{F(A \cdot B)} = \mu_{F_A} \cdot \mu_{F_B} \tag{2}$$

Definition 2: Given two fuzzy events, A and B, with $\mu_{F_A}(\cdot)$ and $\mu_{F_B}(\cdot)$ being the membership functions of event A and event B, respectively. The conditional probability of fuzzy event A given a fuzzy event B is defined by (Bhattacharyya, 1998) as

$$Prob[A|B] = \frac{prob[A \cdot B]}{Prob[B]}; Prob[B] > 0 \tag{3}$$

2.1 *Between-States Transition Probabilities for Non-Homogeneous Markov Fuzzy Manpower System*

LetX_t and X_t^f denote non-fuzzy and fuzzy states of the manpower system at time, t respectively. For X_{t_i} define $n_i(t) = \sum_{j=1}^{k} n_{ij}(t)$ to be the number of personnel in category, G_i, at time, t, where the observed flow, $n_{ij}(t)$, denotes the number of personnel in category G_i at time, t that would be promoted to category $G_j(i, j = 1, 2, \ldots, k)$ at time $t + 1$. Define $a_{ij}(t) = prob(X_{t+1} = G_j/X_t = G_i)$ to be the probability that a member belonging to personnel grade, G_i, at time, t, would be promoted to grade $G_j(i, j = 1, 2, \ldots, k)$ at time, t+1. For the non-homogenous Markov chain $a_{ij}(t)$, the maximum likelihood estimate can be computed as a_{ij}

$$a_{ij}(t) = \frac{\sum_t n_{ij}(t)}{\sum_t n_j(t)} \tag{4}$$

For X_t^f, let $P(F, t)$ denote a 4 x 4 probability matrix of transitions between the fuzzy states, F_r. The element, $P_{F_rF_s}(t)$, of the matrix, $P(F, t)$, represents the probability that a personnel in category, G_i, with a particular level of combination of the personnel traits at time, t, would possess or move to another level of combination at time, t+1 . It can be calculated as follows;

$$p_{F_rF_s}(t) = Prob[X_{t+1}^f = F_s/X_t^f = F_r] = \frac{prob[X_{t+1}^f = F_s, X_t^f = F_r]}{prob[X_t^f = F_r]} \tag{5}$$

$$prob[X^f_{t+1} = F_s, X^f_t = F_r] = \sum_{i=1}^{k}\sum_{j=1}^{k} prob[X_{t+1} = G_j, X_t = G_i]\mu_{F_rF_s}(i,j)$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} prob[X_{t+1} = G_j/X_t = G_i]prob[X_t = G_i]\mu_{F_r}(i)\mu_{F_s}(j)$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)prob[X_t = G_i]\mu_{F_r}(i)\mu_{f_s}(j) \quad (6)$$

Again,

$$prob[X^f_t = F_r] = \sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i) \quad (7)$$

$$p_{F_rF_s}(t) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)prob[X_t = G_i]\mu_{F_r}(i)\mu_{F_s}(j)}{(\sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i))}$$

$$p_{F_rF_s}(t) = (K_{F_r}(t))^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)prob[X_t = G_i]\mu_{F_r}(i)\mu_{F_s}(j) \quad (8)$$

Where $K_{F_r}(t) = \sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i)$.

2.2 *Wastage Probabilities for Non-Homogeneous Markov Fuzzy Manpower System*

Let 0 demote the external environment to which a member who leaves the system is transferred. Let $P_{i0}(t)$ be the probability that a member who leaves the system at time $t+1$ was a member of G_i at time t. Then

$$P_{i0}(t) = \text{prob[member leaves manpower system at time } t+1/X_t = G_i] \quad (9)$$

Similarly,

$$P_{F_r0}(t) = \text{prob[member leaves the manpower system at time } t+1/X^f_t = F_r]$$

$$P_{F_r0}(t) = \frac{\text{prob[member leaves the system at time } (t+1), X_t^f = F_r]}{prob[X_t^f = F_r]} \quad (10)$$

$$\begin{aligned}
&\text{prob[member leaves the system at } (t+1), X_t^f = F_r] \\
&= \sum_{j=1}^{k} \text{prob[member leaves at } (t+1), X_t = G_i]\mu_{F_r}(i) \\
&= \sum_{i=1}^{k} \text{prob[member leaves the system at time } (t+1)/X_t = G_i] \\
&\times prob[X_t = G_j]\mu_{F_r}(i)
\end{aligned}$$

$$\text{prob[member leaves at time } (t+1)\,, X_t^f = F_r] = \sum_{i=1}^{k} p_{i0}(t)\text{prob}[X_t = G_i]\mu_{F_r}(i) \quad (11)$$

$$\text{prob}[X_t^f = F_r] = \sum_{i=1}^{k} \text{prob}[X_t = G_i]\mu_{F_r}(i) \quad (12)$$

$$p_{F_r0}(t) = \frac{\sum_{i=1}^{k} p_{i0}(t)prob[X_t = G_i]\mu_{F_r}(i)}{\sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i)}$$

$$P_{F_r0}(t) = (K_{F_r}(t)^{-1} \sum_{i=1}^{k} p_{i0}(t)\text{prob}[X_t = G_i]\mu_{F_r}(i) \quad (13)$$

2.3 *Recruitment Probabilities for the Non-Homogeneous Markov Fuzzy Manpower System*

Considering that the individuals (new entrants) are recruited from the exsternal environment, 0, into the manpower system in time period, t,

$$LetP_{0j}(t) = prob[X_t = G_j/\text{new member is recruited into the system]}; \quad (14)$$

Similarly,

$$p_{0F_s}(t) = prob[X_t^f = F_s/\text{new member is recruited into the system]} \quad (15)$$

$$'P_{0F_s}(t) = \frac{prob[X_t^f = F_s, \text{new member is recruited into the system}]}{\text{prob[new member is recruited into the system]}} \quad (16)$$

But

$prob[X_t^f = F_s,$ new member is recruited into the system]

$$= \sum_{j=1}^{k} prob[X_t = G_j, \text{new member is recruited into the system}]\mu_{F_r}(j)$$

$$= \sum_{j=1}^{k} prob[X_t = G_j/\text{new member is recruited into the system}]x$$

$$\text{prob[new member is recruited into the system]}\mu_{F_r}(j) \quad (17)$$

$$p_{0F_s}(t) = \sum_{j=1}^{k} p_{0j}(t)\mu_{F_s}(j). \quad (18)$$

2.4 *Total Transition Probability Matrix of the Non-Homogeneous Markov Fuzzy Manpower System*

Let $Q(t) = \begin{bmatrix} q_{11}(t) & q_{12}(t) & \dots & q_{1k}(t) \\ q_{21}(t) & q_{22}(t) & \dots & q_{2k}(t) \\ \dots & qi,j & \dots & \dots \\ q_{k1}(t) & q_{k2}(t) & \dots & q_{kk}(t) \end{bmatrix}$,
where the (i, j)-elements $(q_{ij}(t))$ of the matrix is given by

$$q_{ij}(t) = a_{ij}(t) + p_{i0}(t)p_{0j}(t) \quad (19)$$

The $q_{ij}(t)$ expresses the total probability that either a member who is in (crisp) state, G_i, is promoted to state, G_j, $(p_{ij}(t))$, or a member that is in state, G_i, leaves the system, $(p_{i0}(t))$, and a new member is recruited and allocated to state, $G_j(P_{0j}(t))$.

Similarly, let $Q_f(t) = \begin{bmatrix} q_{F_1F_1}(t) & q_{F_1F_2}(t) & q_{F_1F_3}(t) & q_{F_1F_4}(t) \\ q_{F_2F_1}(t) & q_{F_2F_2}(t) & q_{F_2F_3}(t) & q_{F_2F_4}(t) \\ q_{F_3F_1}(t) & q_{F_3F_2}(t) & q_{F_3F_3}(t) & q_{F_3F_4}(t) \\ q_{F_4F_1}(t) & q_{F_4F_2}(t) & q_{F_4F_3}(t) & q_{F_4F_4}(t) \end{bmatrix}$ represent the total transition probability matrix of the non-homogeneous Markov manpower system with the four fuzzy states, the elements of the matrix which are given by;

$$q_{F_rF_s}(t) = p_{F_rF_s}(t)p_{F_r0}(t)p_{0F_s}(t) \tag{20}$$

$$q_{F_rF_s}(t) = (K_{F_r}(t))^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)prob[X_t = G_i](\mu_{F_r}(i)\mu_{F_s}(j) \tag{21}$$

$$+((K_{F_r}(t))^{-1}\sum_{i=1}^{k} p_{i0}(t)prob[X_t = G_i]\mu_{F_s}(i))(\sum_{i=1}^{k} p_{0j}(t)\mu_{F_r}(j))$$

$$= (K_{F_r}(t))^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} \mu_{F_r}(i)\mu_{F_s}(j)(a_{ij}(t) + p_{i0}(t)p_{0j}(t))prob[X_t = G_i].$$

$$q_{F_rF_s}(t) = (K_{F_r}(t))^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} q_{ij}(t)prob[X_t = G_i]\mu_{F_r}(i)\mu_{F_s}(j) \tag{22}$$

However, there are known and important connections between $Q(t)$ and $Q_f(t)$. It has been shown in (Bhattacharyya, 1968) that if Markov chain associated with the process of non-fuzzy states are irreducible, then, the corresponding Markov chain associated with the fuzzy states are also irreducible. That is, if $Q(t)$ is irreducible, then $Q_f(t)$ is also irreducible. Thus, in a similar way but extending to the notion to the case of non-homogeneous Markov manpower system where transition probabilities of members are assumed to be dependent on time, we have

$$Q_f(t) = \omega_1(t)\Pi'\omega_2(t)Q(t)\Pi \tag{23}$$

where $\omega_1(t) = diag(\theta_{1r}(t))$, with, $Q_{1r}(t) = (\Sigma_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i))^{-1}$ and $\omega_2(t) = diag(\theta_{2r(t)})$ with $\theta_{2r}(t) = prob[X_t = G_i]$.
By letting the row vector $N_r(t) = [N_1(t), N_2(t), N_3(t), N_4(t)]$ represent the population structure concerning the non-homogeneous manpower system with fuzzy states at time, t, the expected population structure at time, $t+1$, $(N_r(t+1))$, can be estimated by the relation

$$N_r(t+1) = Q_f(t)N_r'(t) + \Delta T(t)P_{0F}'(t) \tag{24}$$

2.5 *State Probability Vector for The manpower System*

Theorem 1 (Symeonaki, 2017): If $\{A\}_{t=1}^{\infty}$ is a sequence of irreducible, regular stochastic matrices, and $\lim_{t\to\infty} A(t) = A$, then the product $\Pi_{i=k}^{t} A(i)$ converges to an irreducible regular stable matrix,A^*, which is $A^* = \lim_{t\to\infty} A^t$. Now, assuming that the sequence of embedded matrices,$\{Q(t)\}_{t=1}^{\infty}$,of regular and irreducible stochastic matrix for all t, and that the $\lim_{t\to\infty} Q(t) = Q$, then $Q^* = \lim_{t\to\infty} Q^t$ is a stable stochastic matrix. Similarly, assuming that the sequence of embedded matrices $\{Q_f(t)\}_{t=1}^{\infty}$ of regular and irreducible stochastic matrix for all t for the fuzzy manpower system and that $\lim_{t\to\infty} Q_f(t) = Q_f$, then $Q_f^* = \lim_{t\to\infty} Q_f^t$ is a stable stochastic matrix.

That is, $Q^* = \lim_{t\to\infty} Q^t = \begin{bmatrix} \pi_1 & \pi_2 & \dots & \pi_3 \\ \pi_1 & \pi_2 & \dots & \pi_3 \\ \dots & \dots & \dots & \dots \\ \pi_1 & \pi_2 & \dots & \pi_3 \end{bmatrix}$

where $\pi = \pi Q$

$p^{(t)} = prob[X_t = G_i] \implies p^{(t)} = p^{(1)}Q(1,t) = P^{(t)} = p^{(1)}\Pi_{i=1}^{t} Q(i)$

Thus, $\lim_{t\to\infty} p(t) = p(1)Q^* = p^*$, where p^* represents any row of the matrix, Q^*.

Then, $Q_f = \lim_{t\to\infty} Q_f(t) = \omega_1\Pi'\omega_2 Q\Pi$ and $Q_f^* = \lim_{t\to\infty} Q_f^t$ where,$\omega_1 = diag(\theta_{1r})$, with, $\theta_{1r} = (\Sigma_{i=1}^{k} p_i^* \mu_{F_r}(i))^{-1}$, where p_i^* are the $i - th$ elements of the vector,$p^*, \omega_2 = diag(\theta_{2r})$ with $\theta_{2r} = p_i^*$.

3. Results and Discussion

Data below are personnel flow (recruitment (R_{0j}), promotion (a_{ij}), and wastage (G_6) flows) for the organization, Satajanus Nigeria Limited, Port Harcourt from the period 2018 to 2022, where $G_1, G_2, \dots, G_5$ represents (1) Sales associates (2) departmental managers (3) section managers (4) assistant store managers (5) Store managers; and $n_i(t)$ is the total number of individual in each category at time, t.(see appendix 1). This is implemented using MATLAB. From appendix 1, we have

$$Q = \lim_{t\to\infty} Q(t) = \begin{bmatrix} 0.6781 & 0.2654 & 0.0205 & 0.0205 & 0.0155 \\ 0.0391 & 0.6543 & 0.2839 & 0.0130 & 0.0097 \\ 0.0549 & 0.0412 & 0.6037 & 0.2865 & 0.0137 \\ 0.0429 & 0.0321 & 0.0142 & 0.6143 & 0.2964 \\ 0.0726 & 0.0544 & o.2042 & 0.0242 & 0.8246 \end{bmatrix}$$

$$Q^* = \lim_{t\to\infty} Q^t = \begin{bmatrix} 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \end{bmatrix}$$

The matrix converged at the 7th iteration. The matrix,Π is estimated based on the knowledge that experts possess on system under consideration. That is, the assignment of membership values to fuzzy states is based on previous studies concerning the influence of individual traits on job performance. Studies have found positively significant association between openness and conscientiousness on individual innovativeness, (Ali, 2017). It was found that people who have higher level of openness and conscientiousness are more innovative as compared to those having low levels. Thus, it is seldom does an employee who possesses advance level of openness and advance level of conscientiousness results in low levels of job performance and innovative capability or low level of career development. Using the aforementioned experts' knowledge on manpower systems behavior, we have

$$\Pi = \begin{bmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.5 & 0.3 & 0.2 & 0 \\ 0.1 & 0.4 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

To estimate elements of ω_1, we have, $\theta_{11} = (\Sigma_{i=1}^{5} p_i^* \mu_{F_1}(i))^{-1} = (0.1291 * 0.7 + 0.1851 * 0.5 + \ldots + 0.3335 * 0)^{-1} = 4.5838$
Others are similarly obtained

$$\omega_1 = \begin{bmatrix} 4.5838 & 0 & 0 & 0 \\ 0 & 4.4783 & 0 & 0 \\ 0 & 0 & 4.8063 & 0 \\ 0 & 0 & 0 & 2.8524 \end{bmatrix}$$

$$\omega_2 = \begin{bmatrix} 0.1291 & 0 & 0 & 0 & 0 \\ 0 & 0.1851 & 0 & 0 & 0 \\ 0 & 0 & 0.1906 & 0 & 0 \\ 0 & 0 & 0 & 0.1618 & 0 \\ 0 & 0 & 0 & 0 & 0.3335 \end{bmatrix}$$

$$Q_f = \lim_{t\to\infty} Q_f(t) = \begin{bmatrix} 0.4385 & 0.2724 & 0.2105 & 0.0786 \\ 0.2414 & 0.2630 & 0.2483 & 0.2473 \\ 0.1949 & 0.2531 & 0.2534 & 0.2986 \\ 0.0800 & 0.1497 & 0.1540 & 0.6163 \end{bmatrix}$$

$$Q_f^* = \lim_{t\to\infty} Q_f^t = \begin{bmatrix} 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \end{bmatrix}$$

From Q_f, the estimated value of probability of transition, $q_{F_1F_4} = 0.0786$, represents the probability that a personnel in the system who possessed naive levels of both openness and conscientiousness, at time, t, would possess the traits, openness and conscientiousness, both at advanced level at time, $t+1$ and so on.

And directly associated with Q_f is the estimated matrix, Π of the fuzzy membership function, where the membership value $\mu_{F_1}(1) = 0.7$ corresponds to the degree to which the concept of employee's possession of naive level of openness and naive level of conscientiousness (denoted by F_1) is compatible with sales associate category (denoted by G_1), The steady state probability for the fuzzy manpower system is obtained as $[0.2181, 0.2233, 0.20280, 0.3505]$.$0.2181$ is the probability of remaining in fuzzy state F_1 (that is naive level of openness and naive level of conscientiousness) etc. The result suggests that greater proportion of staff would possess advance level of openness and advance level of conscientiousness in the long run compared to other levels of combinations of personnel traits.

4. Conclusion

In manpower planning analyses, it is assumed that every personnel belonging in a particular homogeneous group possesses homogeneous transition behavior. However, in what looks like a scenario which clearly leads to deviation from the homogeneity assumption, most organizations based their promotion (transition) requirements on innovative capability and job performance level This results in individuals having different promotion behaviors even though belonging in the same group, since they possess different personality traits which influence their innovativeness and productivity in different ways. In order to incorporate personality traits as well as tackling the problem of ambiguity associated with gradual promotion of employees between the crisp states of a manpower system, the methodology proposed in this study is recommended. However, this paper failed to address a situation where more than two types of personality traits and more than two linguistic variables are incorporated in a fuzzy non-homogeneous Markov manpower model. This constitutes future research.

References

Agboola, S. O. and Ahmad, H. I. (2023). On the Markov Chain Prediction of Academic Manpower System in Private University in Nigeria. FUW Trends in Science and Technology Journal, 8(2), 308–318.

Ali, I. (2017). Personality traits, individual innovativeness and satisfaction with life. Journal and Innovation and Knowledge, 4, 38–46.

Assi, P. N. and Effanga, E. O. (2021). Optimal Manpower Recruitment and Promotion Policies for the Finitely Graded System using Dynamic Programming. Heliyon, 7(2021) e07424. https://doi.org/10.1016/j.heliyon.2021 e07424

Bartholomew, D. J., Forbes, A. F. and McClean, S. I. (1991). Statistical Techniques for Manpower planning (2nd edition), Chichester: Wiley Publishers.

Bhattacharyya, M. (1998). Fuzzy Markovian decision process, Fuzzy Sets and System, 99(1998), 273–282.

Belhaj, R. and Tkiouat, M. (2013). A Markov model for human resources supply forecast; dividing the HR system into subgroups. Journal of Service Science and Management, 6, 211–217. http://dx.doi.org/10.4236/jssm.2013.63023

DeFeyter, T. (2006). Modelling heterogeneity in manpower planning: dividing the personnel system into more homogeneous subgroups. Applied Stochastic Models in Business and Industry, 22, 321- 334. doi:10.1002/asmb10

Ekhosuehi, V. U. and Osagiede, A. A. (2006). Application of Markovian Model to School

Enrolment Projection Process. Global Journal of Mathematical Sciences, 5(1), 9-16.

Ekhosuehi, V. U. Enagbonma, O. and Osagiede, A.A, (2017). On Stable Growth Index in Graded Structured Manpower System. Journal of Nigeria Statistical Association, 29, 1–13.

Ezugwu, V. O. and Ologun, S. (2017). Markov chain: A Predictive Model for Manpower Planning. Journal of Applied Science and Environmental Management, 21(3), 557–565.

Ezugwu, V. O. and Igbinosun, L. I. (2016). Poetfolio Allocation Under Vendor Managed Inventory; Markov Decision Process. Journal of Sci, Environ. Manage, 20(14), 1127–1135.

Ezugwu, V. O. and Igbinosun, L. I. (2020). Analysis of Manpower System Using Multi Absorbing States Markov Chain. International Journal of Statistics and Applied Mathematics, 5(2), 92–99.

Ezugwu, V. O., Ologun, S. and Anietimg, A. E. (2013). Markov Chain: A Stabilizer to Market Equilibrium Mix. International Journal of Advanced Scienti
c and Technical Research, 3(1), 565–575.

Guerry, M. A. (2011). Hidden Heterogeneity in Manpower Systems: a Markov-switching Model Approach. European Journal of Operational Research, 210, 106–113.

Jeeva, M. and Geetha, N. (2013). Recruitment Model in Manpower Planning Under Fuzzy Environment, British Journal of Applied Science and Technology, 3(4), 1380–1390.

Ossai, E. O. (2023). Steady State Conditions in Tractable Markov Manpower Model for an Extended Manpower System. Asian Journal Probability and Statistics, 25(1), 48–56.

Symeonaki, M. (2017). Rate of Convergence in Fuzzy Non-homogeneous Markov Systems, Communications in Statistics Theory and Methods, DOI: 10.1080/03610926.2017.1395044

Symeonaki, M. A. and Stamou, G. B. (2004). Theory of Markov Systems with Fuzzy States, Fuzzy Sets and Systems, 143, 427–445. doi:10.1016/S0165-0114(03)00016-2

Symeonaki, M. A., Stamou, G. B. and Tzafestas, S. G. (2002). Fuzzy Non-Homogeneous Markov Systems, Applied Intelligence, 17, 203–214.

Udom, A. U. and Ebedoro, U. G. (2019). On the Multinomial Hidden Markov Model for Hierarchical Manpower Systems. Communications in Statistics- Theory and Methods, https://doi.org/10.1080/03610926.2019.1650185

Ugwuowo, F. I. and McClean, S. I. (2000). Modelling Heterogeneity in a Manpower System: A Review. Applied Stochastic Models in Business and Industry, 16, 99–110.

Vassiliou, P. G. (1982). Asymptotic Behaviour of Markov Systems. Journal of Applied Probability, 19, 851–857.

Vassiliou, P. G. (2018). Laws of Large Numbers for Non-Homogeneous Markov Systems, Methodology and Computing in Applied probability, https://doi.org/10.1007/s11009-017-9612-1

Vassiliou, P. G. (2021). Non-Homogeneous Markov Set System, Mathematics, https/dio.org/10.3390/math9050471.

Vassiliou, P. C. G. (2022). Limiting Distributions of a Non-Homogeneous Markov System in a Stochastic Environment in Continuous Time. Mathemetics, https//doi.org/10.3390/math1008121411

Vassiliou, P. G. (2024). Strong Ergodicity in Non-Homogeneous Markov System with Chronological Order, https://doi.org/10.3390/math12050660

Zadeh, L. A. (1968). Probability Measures of Fuzzy Events. Journal of Mathematical Analysis and Applications, 23, 421–427.

Appendix 1

Table 6; Pooled personnel transition for all the five years

G_i	1	2	3	4	5	G_0	n_i
1	45	16	0	0	0	12	73
2	0	30	13	0	0	5	48
3	0	0	24	11	0	6	41
4	0	0	0	21	10	4	35
5	0	0	0	0	25	6	31
R_{0j}	12	9	4	4	3		32

Appendix 2

MATLAB codes for matrix Q^*

```
% Define your fractional matrix A
Q=[0.6781,0.2654,0.0205,0.0205,.0155;0.0391,0.6543,0.2839,0.0130,.0097;
0.0549,0.0412,0.6037,0.2865,.0137;0.0429,0.0321,0.0142,0.6143,.2964;0.0726,0.0544,
0.0242,0.0242,0.8246];
% Initialize a variable to store the matrix powers
max_power = 1000;
Q_powers = cell(1, max_power);
convergence_threshold = 1e-6;
% Calculate matrix powers
for t = 1:max_power
Q_powers{t} = Q∧t;
% Check for convergence starting from the second power
if t > 1
change = norm(Q_powers{t} - Q_powers{t-1}, 'fro');
if change <convergence_threshold
disp(['Converged at iteration ' num2str(t)]);
break;
end
end
end
disp('Converged Matrix:');
disp(Q_powers{t});
```

Appendix 3

MATLAB codes for matrix Q_f^*

```
% Define your fractional matrix Qf
Qf=[0.4385,0.2724,0.2105,0.0786;0.2414,0.2630,
0.2483,0.2473;0.1949,0.2531,0.2534,0.2986;0.0800,0.1497,0.1540,0.6163];
% Initialize a variable to store the matrix powers
max_power = 1000;
Qf_powers = cell(1, max_power);
convergence_threshold = 1e-6;
% Calculate matrix powers
for t = 1:max_power
Qf_powers{t} = Qf∧t;
% Check for convergence starting from the second power
if t > 1
change = norm(Qf_powers{t}- Qf_powers{t-1}, 'fro');
if change <convergence_threshold
disp(['Converged at iteration ' num2str(t)]);
```

```
break;
end
end
end
disp('Converged Matrix:');
disp(Q_f_powers{t});
```